MATH NOTES

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Chapter 1.

MATRICES AND DETERMINANTS

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Introduction:

The matrices and determinants are used in the field of Mathematics, Physics, Statistics, Electronics and other branches of science. The Matrices have played a very important role in this age of computer science. The idea of matrices was given by the Arthur Cayley, an English Mathematician of 19th century, who first developed, *Theory of Matrices*" "in 1858.

Matrix:

"An arrangement of different elements in the rows and columns, within square brackets is called Matrix".

e.g $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

The real numbers used in the formation of the matrix are called entries or elements of the matrix. The matrices are denoted by the capital letters $A, B, C, D, \dots, M, N$ *etc.* of the English alphabets.

Rows and Columns of a Matrix:

In a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, the entries presented in the horizontal way are called rows.

In a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, the entries presented in the vertical way are called columns.

Order of a Matrix:

Order of Matrix tells us about no of rows and columns.

$\boldsymbol{Order\ of\ a\ matrix = no.of\ rows \times no.of\ columns.}$

If a matrix A has m rows and n column then its order is

$$\boldsymbol{O(A) = m \times n\ or\ m - by - n.}$$

For example

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix}\ has\ order\ 3 - by - 3\ or\ 3 \times 3.$$

Equal matrices:

"Two matrices are said to be equal if

i. **The order of matrix A =The order of Matrix B**

ii. **Their corresponding elements are equal. Thus**

$$\boldsymbol{A = B.}"$$

Example:

$$A = \begin{bmatrix} 1 & 2 \\ -5 & 7 \end{bmatrix}\ and\ B = \begin{bmatrix} 1 & 1+1 \\ -5 & 5+2 \end{bmatrix}$$

are equal matrices.

Exercise 1.1

Question.1.Find the order of the following matrices.

(i). $A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$

Solution.

$\boldsymbol{Order\ of\ A = O(A)}$ **=2-by-2** $\boldsymbol{or\ 2 \times 2}$

(ii). $B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

Solution.

$\boldsymbol{Order\ of\ B = O(B)}$ **=2-by-2**$\boldsymbol{or\ 2 \times 2}$

(iii). $C = [2 \quad 4]$

Solution.

$\boldsymbol{Order\ of\ C = O(C) =}$ **1-by-2** $\boldsymbol{or\ 1 \times 2}$

(iv). $D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$

Solution.

$\boldsymbol{Order\ of\ D = O(D)}$ **=3-by-1**$\boldsymbol{or\ 3 \times 1}$

(v). $E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

Solution.

$\boldsymbol{Order\ of\ E = O(E)}$ **=3-by-2** $\boldsymbol{or\ 3 \times 2}$

(vi). $F = [2]$

Solution.

$\boldsymbol{Order\ of\ F = O(F)}$ **=1-by-1**$\boldsymbol{or\ 1 \times 1}$

(vii). $G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

Solution.

$\boldsymbol{Order\ of\ G = O(G)}$ **=3-by-3**$\boldsymbol{or\ 3 \times 3}$

(viii). $H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Solution.

$\boldsymbol{Order\ of\ H = O(H)}$ **=2-by-3** $\boldsymbol{or\ 2 \times 3}$

Question.2. which of the following matrices are equal?

$A = [3]$, $B = [3 \quad 5]$

$C = [5-2]$, $D = [5 \quad 3]$

$E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, $F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$,

$G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$ $H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$,

$I = [3 \quad 3+2]$, $J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Solution.

From above matrices

$$A = C$$
$$E = H = J$$
$$F = G$$

Question.3. Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Solution.
Given
$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$
By the definition of equal matrices, we have
$$a+c=0 \to (i)\ , a+2b=-7 \to (ii)\ ,$$
$$c-1=3 \to (iii)\ , 4d-6=2d \to (iv)$$
From (iii), we have
$$c=3+1=4$$
From (iv), we have
$$4d-6=2d$$
$$4d-2d=6$$
$$2d=6$$
$$d=\frac{6}{2}$$
$$d=3$$
Using value of $c=4$ in (i), we have
$$a+4=0$$
$$a=-4$$
Using value of $a=-4$ in (ii), we have
$$-4+2b=-7$$
$$2b=-7+4$$
$$2b=-3$$
$$b=-\frac{3}{2}$$
Hence $a=-4\ , b=-\frac{3}{2}\ , c=4$ and $d=3$.

Types of Matrices:

Row matrix:

"A matrix having single row is called Row Matrix."

Example:

$M=[1\ \ 2\ \ 3]$ is a row matrix of order $1-by-3$.

Column matrix:

A matrix having single column is called column Matrix.

Example:
$$M=\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} \text{ is a column matrix of order } 3-by-1.$$

Rectangular matrix:

A matrix in which number of rows is not equal to number of columns is called rectangular Matrix.

Example:
$$\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix} \text{ are rectangular matrices.}$$

Square matrix:

"A matrix in which number of rows is equal to the number of columns then matrix is called square matrix."

Example:
$$A=\begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix} \text{ has order } 3-by-3.$$

Null or Zero Matrix:

"A matrix whose each element is zero, is called a null or zero matrix. It is denoted by O."

Examples:
$$[0]\ ,\ [0\ \ 0]\ ,\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ are null matrices.}$$

Transpose of a Matrix:

"A matrix obtained by changing the rows into columns or columns into rows of a matrix is called transpose of that matrix. If A is a matrix, then its transpose matrix is denoted by A^t."

Example:
$$\text{If } A=\begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix} \text{ then } A^t=\begin{bmatrix} 1 & 9 & 4 \\ 2 & 7 & 6 \\ 3 & 6 & 8 \end{bmatrix}$$
$$\text{If } B=\begin{bmatrix} 1 & 3 & 2 \\ 1 & 9 & 4 \end{bmatrix} \text{ then } B^t=\begin{bmatrix} 1 & 1 \\ 3 & 9 \\ 2 & 4 \end{bmatrix}$$
If a matrix B is of order 2-by-3 then order its transpose matrix B^t is 3-by-2.

Negative of a Matrix:

"Let A be a matrix. Then its negative, $-A$ is obtained by changing the signs of all the entries of A."

Example:
$$\text{If } A=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \text{ then } -A=\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}.$$

Symmetric matrix:

"Let A be the square matrix, if $A^t=A$ then A is called symmetric matrix."

Example:
$$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix} \text{ is a square matrix then}$$
$$A^t=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix}=A.$$
$$\text{Thus } A \text{ is symmteric matrix.}$$

Skew-symmetric matrix:

"Let A be the square matrix, if $A^t=-A$ then A is called skew symmetric matrix."

Example:
$$A=\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} \text{ is a square matrix then}$$
$$A^t=\begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}=-\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$$
$$=-A.$$
$$\text{Thus } A \text{ is a skew}-\text{symmteric matrix.}$$

Diagonal matrix:

"A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not

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zero and non-diagonal entries are zero.”

Example:

$$A=\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix},B=\begin{bmatrix}2&0\\0&3\end{bmatrix}\ and\ C=\begin{bmatrix}1&0\\0&0\end{bmatrix}$$

$are\ called\ diagonal\ matrices.$

Scalar Matrix:

“A diagonal matrix having same elements in principle diagonal except 1 or 0 is called scalar matrix.”

Example:

$$A=\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}\ and\ B=\begin{bmatrix}3&0\\0&3\end{bmatrix}\ are\ Scalar\ matrices.$$

Unit Matrix or Identity Matrix:

A diagonal matrix is called identity matrix if all diagonal entries are 1. It is denoted by I.

$$I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\ and\ I=\begin{bmatrix}1&0\\0&1\end{bmatrix}\ are\ identity\ matrices.$$

Exercise 1.2

Question.1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

$$\boldsymbol{A}=\begin{bmatrix}\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}\end{bmatrix},\quad \boldsymbol{B}=[\mathbf{2}\quad\mathbf{3}\quad\mathbf{4}],\quad \boldsymbol{C}=\begin{bmatrix}\mathbf{4}\\\mathbf{0}\\\mathbf{6}\end{bmatrix}$$

$$\boldsymbol{D}=\begin{bmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{0}&\mathbf{1}\end{bmatrix},\quad \boldsymbol{E}=[\mathbf{0}],\quad \boldsymbol{F}=\begin{bmatrix}\mathbf{5}\\\mathbf{6}\\\mathbf{7}\end{bmatrix}$$

Solution.

Identity Matrices: D

Row Matrices: $B\ and\ E.$

Column Matrices: $C\ ,E\ and\ F.$

Null Matrices: $A\ and\ E.$

Question.2. From the following matrices, identify (a) Square matrices, (b) Rectangular matrices, (c) Row matrices, (d) Column matrices, (e) Identity Matrices, (f) Null matrices.

(i).
$$\begin{bmatrix}\mathbf{-8}&\mathbf{2}&\mathbf{7}\\\mathbf{12}&\mathbf{0}&\mathbf{4}\end{bmatrix}\quad (\boldsymbol{ii}).\begin{bmatrix}\mathbf{3}\\\mathbf{0}\\\mathbf{1}\end{bmatrix}\quad (\boldsymbol{iii}).\begin{bmatrix}\mathbf{6}&\mathbf{-4}\\\mathbf{3}&\mathbf{-2}\end{bmatrix}\quad (\boldsymbol{iv}).\begin{bmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{0}&\mathbf{1}\end{bmatrix}\quad (\boldsymbol{v})\begin{bmatrix}\mathbf{1}&\mathbf{2}\\\mathbf{3}&\mathbf{4}\\\mathbf{5}&\mathbf{6}\end{bmatrix}$$

(vi). $[\mathbf{3}\quad\mathbf{10}\quad\mathbf{-1}]$ **(vii).**
$$\begin{bmatrix}\mathbf{1}\\\mathbf{0}\\\mathbf{0}\end{bmatrix}\quad (\boldsymbol{viii}).\begin{bmatrix}\mathbf{1}&\mathbf{2}&\mathbf{3}\\\mathbf{-1}&\mathbf{2}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{1}\end{bmatrix}\quad (\boldsymbol{ix}).\begin{bmatrix}\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}\end{bmatrix}$$

Solution.

(a). Square Matrices: (iii). (iv). (viii).

(b). Rectangular Matrices: (i). (ii). (v).

(c). Row Matrices: (vi).

(d). (ii). (vii).

(e). (iv).

(f). (ix).

Question.3. From the following matrices, identify Diagonal matrices, Scalar matrices and Unit (identity) matrices.

$$\boldsymbol{A}=\begin{bmatrix}\mathbf{4}&\mathbf{0}\\\mathbf{0}&\mathbf{4}\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}\mathbf{2}&\mathbf{0}\\\mathbf{0}&\mathbf{-1}\end{bmatrix},\quad \boldsymbol{C}=\begin{bmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{0}&\mathbf{1}\end{bmatrix}$$

$$\boldsymbol{D}=\begin{bmatrix}\mathbf{3}&\mathbf{0}\\\mathbf{0}&\mathbf{0}\end{bmatrix},\quad \boldsymbol{E}=\begin{bmatrix}\mathbf{5-3}&\mathbf{0}\\\mathbf{0}&\mathbf{1+1}\end{bmatrix}$$

Solution.

Diagonal Matrices: $A\ ,B\ ,C\ ,D\ ,E.$

Scalar Matrices: $A, C, E.$

Unit Matrices: $C.$

Question.4. Find the negative of matrices A, B, C, D and E when:

(i). $\boldsymbol{A}=\begin{bmatrix}\mathbf{1}\\\mathbf{0}\\\mathbf{-1}\end{bmatrix}$

Solution.

$$-A=\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

(ii). $B=[5\quad 1\quad -6]$

Solution.

$$-B=[-5\quad -1\quad 6]$$

(iii). $C=\begin{bmatrix}2&3\\0&5\end{bmatrix}$

Solution.

$$-C=\begin{bmatrix}-2&-3\\0&-5\end{bmatrix}$$

(iv). $D=\begin{bmatrix}2&3\\-4&5\end{bmatrix}$

Solution.

$$-D=\begin{bmatrix}-2&-3\\4&-5\end{bmatrix}$$

(v). $E=\begin{bmatrix}1&2\\3&4\end{bmatrix}$

Solution.

$$-E=\begin{bmatrix}-1&-2\\-3&-4\end{bmatrix}$$

Question.5. Find the transpose of the following matrices:

(i). $\boldsymbol{A}=\begin{bmatrix}\mathbf{0}\\\mathbf{1}\\\mathbf{-2}\end{bmatrix}$

Solution.

$$\boldsymbol{A^t}=[\mathbf{0}\quad\mathbf{1}\quad\mathbf{-1}]$$

(ii). $\boldsymbol{C}=\begin{bmatrix}\mathbf{1}&\mathbf{2}\\\mathbf{2}&\mathbf{-1}\\\mathbf{3}&\mathbf{0}\end{bmatrix}$

Solution.

$$C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$$

(iii). $D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

Solution.

$$D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

(iv). $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$

Solution.

$$E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$$

(v). $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution.

$$F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Question.6. Verify that if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$, *then*

(i). $(A^t)^t = A$

Solution.

Given

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = A$$

$$(A^t)^t = A$$

Hence Proved.

(ii). $(B^t)^t = B$

Solution.

Given

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$$

$$(B^t)^t = B$$

Hence Proved

Addition of matrices:

"Let A and B be any two matrices of same order then A and B are comfortable for addition." Addition of A and B, Written as $A + B$ is obtained by adding the entries of the matrix A to the corresponding entries of the matrix B."

Example:

$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix}$ $and\ B = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ $are\ comfortable\ for\ addition.$

$$A + B = \begin{bmatrix} 2-2 & 3+3 & 0+4 \\ 1+1 & 0+2 & 6+3 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 4 \\ 2 & 2 & 9 \end{bmatrix}$$

Subtraction of matrices:

Let A and B be any two matrices of same order then A and B are comfortable for Subtraction.
Subtraction of A and B, Written as $A - B$ is obtained by subtracting the entries of the matrix A to the corresponding entries of the matrix B.

Example:

$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix}$ $and\ B = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$

$are\ comfortable\ for$ Subtraction.

$$A - B = \begin{bmatrix} 2+2 & 3-3 & 0-4 \\ 1-1 & 0-2 & 6-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -4 \\ 0 & -2 & 3 \end{bmatrix}$$

Multiplication of a Matrix by a Real Number:

Let A be any matrix and the real number k be a scalar. Then the scalar multiplication of matrix A with k is obtained by multiplying each entry of matrix A with k. It is denoted by kA.

Example:

$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix}$ $then\ kA = \begin{bmatrix} 2k & 3k & 0 \\ 1k & 0 & 6k \end{bmatrix}$

Commutative Law for Addition.

If A and B are two matrices of the same order, Then $A + B = B + A$ *is* called commutative law under addition.

$$A + B = B + A$$

Associative Law for Addition:

If A, B and C are three matrices of the same order, Then $(A + B) + C = A + (B + C)$ *is* Called Associative law under addition.

$$(A + B) + C = A + (B + C)$$

Additive Identity of a Matrix:

If A and B are two matrices of same order and $A + B = A = B + A$

Then matrix B is called additive identity of matrix A.
For any matrix A and zero matrix of same order, O is called additive identity of A as

$$A + O = A = O + A.$$

Additive Inverse of a Matrix:

If A and B are two matrices of same order and $A + B = O = B + A$

Then matrix B is called additive inverse of matrix A.
"Additive inverse of any matrix A is obtained by changing to negative of the symbols (entries) of each non zero entry of A."

Exercise 1.3

Question.1. which of the following matrices are comfortable for addition?

$$\boldsymbol{A}=\begin{bmatrix}\mathbf{2}&\mathbf{1}\\\mathbf{-1}&\mathbf{3}\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}\mathbf{3}\\\mathbf{1}\end{bmatrix},\quad \boldsymbol{C}=\begin{bmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{2}&\mathbf{-1}\\\mathbf{1}&\mathbf{-2}\end{bmatrix}$$

$$\boldsymbol{D}=\begin{bmatrix}\mathbf{2+1}\\\mathbf{3}\end{bmatrix},\quad \boldsymbol{E}=\begin{bmatrix}\mathbf{-1}&\mathbf{0}\\\mathbf{1}&\mathbf{2}\end{bmatrix},$$

$$\boldsymbol{F}=\begin{bmatrix}\mathbf{3}&\mathbf{2}\\\mathbf{1+1}&\mathbf{-4}\\\mathbf{3+2}&\mathbf{2+1}\end{bmatrix}$$

Solution.

Since order of $A\ and\ E\ are$ same so they are comfortable for addition.

Also order of $B\ and\ D\ are$ same so they are comfortable for addition.

Also order of $C\ and\ F\ are$ same so they are comfortable for addition.

Question.2. Find the additive inverse of the following matrices:

(i). $\boldsymbol{A}=\begin{bmatrix}\mathbf{2}&\mathbf{4}\\\mathbf{-2}&\mathbf{1}\end{bmatrix}$

Solution.

$$Additive\ inverse\ of\ A=-A=\begin{bmatrix}2&4\\-2&1\end{bmatrix}$$

(ii). $\boldsymbol{B}=\begin{bmatrix}\mathbf{1}&\mathbf{0}&\mathbf{-1}\\\mathbf{2}&\mathbf{-1}&\mathbf{3}\\\mathbf{3}&\mathbf{-2}&\mathbf{1}\end{bmatrix}$

Solution.

$$Additive\ inverse\ of\ B=-B=\begin{bmatrix}-1&0&1\\-2&1&-3\\-3&2&-1\end{bmatrix}$$

(iii). $\boldsymbol{C}=\begin{bmatrix}\mathbf{4}\\\mathbf{-2}\end{bmatrix}$

Solution.

$$Additive\ inverse\ of\ C=-C=\begin{bmatrix}-4\\2\end{bmatrix}$$

(iv). $\boldsymbol{D}=\begin{bmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{-3}&\mathbf{-2}\\\mathbf{2}&\mathbf{1}\end{bmatrix}$

Solution.

$$Additive\ inverse\ of\ D=-D=\begin{bmatrix}-1&0\\3&2\\-2&-1\end{bmatrix}$$

(v). $\boldsymbol{E}=\begin{bmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{0}&\mathbf{1}\end{bmatrix}$

Solution.

$$Additive\ inverse\ of\ E=-E=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$$

(vi). $\boldsymbol{F}=\begin{bmatrix}\sqrt{\mathbf{3}}&\mathbf{1}\\\mathbf{-1}&\sqrt{\mathbf{2}}\end{bmatrix}$

Solution.

$$Additive\ inverse\ of\ F=-F=\begin{bmatrix}-\sqrt{3}&-1\\1&-\sqrt{2}\end{bmatrix}$$

Question.3.If $\boldsymbol{A}=\begin{bmatrix}\mathbf{-1}&\mathbf{2}\\\mathbf{-2}&\mathbf{1}\end{bmatrix},\boldsymbol{B}=\begin{bmatrix}\mathbf{1}\\\mathbf{-1}\end{bmatrix},\boldsymbol{C}=$ $[\mathbf{1}\quad\mathbf{-1}\quad\mathbf{2}],\boldsymbol{D}=\begin{bmatrix}\mathbf{1}&\mathbf{2}&\mathbf{3}\\\mathbf{-1}&\mathbf{0}&\mathbf{2}\end{bmatrix}$

Then find,

(i). $\boldsymbol{A}+\begin{bmatrix}\mathbf{1}&\mathbf{1}\\\mathbf{1}&\mathbf{1}\end{bmatrix}$

Solution.

$$A+\begin{bmatrix}1&1\\1&1\end{bmatrix}=\begin{bmatrix}-1&2\\-2&1\end{bmatrix}+\begin{bmatrix}1&1\\1&1\end{bmatrix}$$

$$=\begin{bmatrix}-1+1&2+1\\-2+1&1+1\end{bmatrix}$$

$$=\begin{bmatrix}0&3\\-1&2\end{bmatrix}$$

Answer.

(ii). $\boldsymbol{B}+\begin{bmatrix}\mathbf{-2}\\\mathbf{3}\end{bmatrix}$

Solution.

$$B+\begin{bmatrix}-2\\3\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix}+\begin{bmatrix}-2\\3\end{bmatrix}$$

$$=\begin{bmatrix}1-2\\-1+3\end{bmatrix}$$

$$=\begin{bmatrix}-1\\2\end{bmatrix}$$

Answer.

(iii). $\boldsymbol{C}+[\mathbf{-2}\quad\mathbf{1}\quad\mathbf{3}]$

Solution.

$$C+[-2\quad 1\quad 3]=[1\quad -1\quad 2]+[-2\quad 1\quad 3]$$

$$=[1-2\quad -1+1\quad 2+3]$$

$$=[-1\quad 0\quad 5]$$

Answer.

(iv). $\boldsymbol{D}+\begin{bmatrix}\mathbf{0}&\mathbf{1}&\mathbf{0}\\\mathbf{2}&\mathbf{0}&\mathbf{1}\end{bmatrix}$

Solution.

$$D+\begin{bmatrix}0&1&0\\2&0&1\end{bmatrix}=\begin{bmatrix}1&2&3\\-1&0&2\end{bmatrix}+\begin{bmatrix}0&1&0\\2&0&1\end{bmatrix}$$

$$=\begin{bmatrix}1+0&2+1&3+0\\-1+2&0+0&2+1\end{bmatrix}$$

$$=\begin{bmatrix}1&3&3\\1&0&3\end{bmatrix}$$

Answer.

(v). $\mathbf{2}\boldsymbol{A}$

Solution.

$$2A=2\begin{bmatrix}-1&2\\2&1\end{bmatrix}$$

$$=\begin{bmatrix}-2&4\\4&2\end{bmatrix}$$

Answer.

(vi). $(\mathbf{-1})\boldsymbol{B}$

Solution.

$$(-1)B=(-1)\begin{bmatrix}1\\-1\end{bmatrix}$$

$$=\begin{bmatrix}-1\\1\end{bmatrix}$$

Answer.

(vii). $(\mathbf{-2})\boldsymbol{C}$

Solution.

$$(-2)C=(-2)[1\quad -1\quad 2]$$

$$=[-1\quad 2\quad -4]$$

Answer.

(viii). $\mathbf{3}\boldsymbol{D}$

Solution.

$$3D=3\begin{bmatrix}1&2&3\\-1&0&2\end{bmatrix}$$

$$=\begin{bmatrix}3&6&9\\-3&0&6\end{bmatrix}$$

Answer.

(ix). $\mathbf{3}\boldsymbol{C}$

Solution.

$$3C=3[1\quad -1\quad 2]$$

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$$= \begin{bmatrix} 3 & -6 & 6 \end{bmatrix}$$

Answer.

Question.4. perform the indicated operations and simplify the following

(i). $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Solution.

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \left(\begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1+1 & 2+1 \\ 3+0 & 1+1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

Answer.

(ii). $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} + \left(\begin{bmatrix} \mathbf{0} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}\right)$

Solution.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 0-1 & 2-1 \\ 3-1 & 0-1 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1-1 & 0+1 \\ 0+2 & 1-1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

Answer.

(iii). $[\mathbf{2} \quad \mathbf{3} \quad \mathbf{1}] + ([\mathbf{1} \quad \mathbf{0} \quad \mathbf{2}] - [\mathbf{2} \quad \mathbf{2} \quad \mathbf{2}])$

Solution.

$$[2 \quad 3 \quad 1] + ([1 \quad 0 \quad 2] - [2 \quad 2 \quad 2])$$
$$= [2 \quad 3 \quad 1] + ([1-2 \quad 0-2 \quad 2-2])$$
$$= [2 \quad 3 \quad 1] + [-1 \quad -2 \quad 0]$$
$$= [2-1 \quad 3-2 \quad 1+0]$$
$$= [1 \quad 1 \quad 1]$$

Answer.

(iv). $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{2} & \mathbf{2} \\ \mathbf{3} & \mathbf{3} & \mathbf{3} \end{bmatrix}$

Solution.

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 0+3 & 1+3 & 2+3 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

Answer.

(vi). $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} \\ \mathbf{3} & \mathbf{1} & \mathbf{2} \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} & \mathbf{-1} \end{bmatrix}$

Solution.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1+1 & 2+0 & 3-2 \\ 2-1 & 3-1 & 1+0 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 2 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

Answer.

(vi). $\left(\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}\right) + \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{bmatrix}$

Solution.

$$\left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
$$= \left(\begin{bmatrix} 1+1 & 2+2 \\ 0+0 & 1+1 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2+1 & 4+1 \\ 0+1 & 2+1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}$$

Answer.

Question5.For the matrices $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{0} \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{-2} & \mathbf{2} \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \end{bmatrix}$,

$\boldsymbol{C} = \begin{bmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} \end{bmatrix}$, **Verify the following rules:**

(i). $\boldsymbol{A} + \boldsymbol{C} = \boldsymbol{C} + \boldsymbol{A}$

Solution.

$$L.H.S = A + C$$
$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$
$$L.H.S = \begin{bmatrix} 1-1 & 2+0 & 3+0 \\ 2+0 & 3-2 & 1+3 \\ 1+1 & -1+1 & 0+2 \end{bmatrix}$$
$$L.H.S = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & -0 & 2 \end{bmatrix}$$
$$R.H.S = C + A$$
$$R.H.S = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$
$$R.H.S = \begin{bmatrix} -1+1 & 0+2 & 0+3 \\ 0+2 & -2+3 & 3+1 \\ 1+1 & 1-1 & 2+0 \end{bmatrix}$$
$$L.H.S = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & -0 & 2 \end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(ii). $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$

Solution.

$$L.H.S = A + B$$
$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$L.H.S=\begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$

$L.H.S=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$

$R.H.S=B+A$

$R.H.S=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$

$R.H.S=\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+1 & 1-1 & 3+0\end{bmatrix}$

$L.H.S=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$

Hence Proved $L.H.S=R.H.S.$

(iii). $B+C=C+B$

Solution.

$L.H.S=B+C$

$L.H.S=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$

$L.H.S=\begin{bmatrix}1-1 & -1+0 & 1+0\\2+0 & -2-2 & 2+3\\3+1 & 1+1 & 3+2\end{bmatrix}$

$L.H.S=\begin{bmatrix}0 & -1 & 1\\2 & -4 & 5\\4 & 2 & 5\end{bmatrix}$

$R.H.S=C+B$

$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$

$R.H.S=\begin{bmatrix}-1+1 & 0-1 & 0+1\\0+2 & -2-2 & 3+2\\1+3 & 1+1 & 2+3\end{bmatrix}$

$R.H.S=\begin{bmatrix}0 & -1 & 1\\2 & -4 & 5\\4 & 2 & 5\end{bmatrix}$

Hence Proved $L.H.S=R.H.S.$

(iv). $A+(B+A)=2A+B$

Solution.

$L.H.S=A+(B+A)$

$L.H.S=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}\right)$

$L.H.S=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+1 & 1-1 & 3+0\end{bmatrix}\right)$

$L.H.S=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$

$L.H.S=\begin{bmatrix}1+2 & 2+1 & 3+4\\2+4 & 3+1 & 1+3\\1+4 & -1+0 & 0+3\end{bmatrix}$

$L.H.S=\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$

$R.H.S=2A+B$

$R.H.S=2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$

$R.H.S=\begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$

$R.H.S=\begin{bmatrix}2+1 & 4-1 & 6+1\\4+2 & 6-2 & 2+2\\2+3 & -2+1 & 0+3\end{bmatrix}$

$R.H.S=\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$

Hence Proved $L.H.S=R.H.S.$

(v). $(C-B)+A=C+(A-B)$

Solution.

$L.H.S=(C-B)+A$

$L.H.S=\left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$

$L.H.S=\left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\0-2 & -2+2 & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$

$L.H.S=\begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$

$L.H.S=\begin{bmatrix}-2+1 & 1+2 & -1+3\\-2+2 & 0+3 & 1+1\\-2+1 & 0-1 & -1+0\end{bmatrix}$

$L.H.S=\begin{bmatrix}-1 & 3 & 2\\0 & 3 & 2\\-1 & -1 & -1\end{bmatrix}$

$R.H.S=C+(A-B)$

$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$

$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1-1 & 2+1 & 3-1\\2-2 & 3+2 & 1-2\\1-3 & -1-1 & 0-3\end{bmatrix}\right)$

$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 3 & 2\\0 & 5 & -1\\-2 & -2 & -3\end{bmatrix}$

$R.H.S=\begin{bmatrix}-1+0 & 0+3 & 0+2\\0+0 & -2+5 & 3-1\\1-2 & 1-2 & 2-3\end{bmatrix}$

$R.H.S=\begin{bmatrix}-1 & 3 & 2\\0 & 3 & 2\\-1 & -1 & -1\end{bmatrix}$

Hence Proved $L.H.S=R.H.S.$

(vi). $2A+B=A+(A+B)$

Solution.

$L.H.S=2A+B$

$$L.H.S = 2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2+1 & 4-1 & 6+1\\4+2 & 6-2 & 2+2\\2+3 & -2+1 & 0+3\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

$R.H.S = A + (A + B)$

$$L.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1!+3 & -1+1 & 0+3\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}1+2 & 2+1 & 3+4\\2+4 & 3+1 & 1+3\\1+4 & -1+0 & 0+3\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(vii). $(\boldsymbol{C} - \boldsymbol{B}) - \boldsymbol{A} = (\boldsymbol{C} - \boldsymbol{A}) - \boldsymbol{B}$

Solution.

$L.H.S = (C - B) - A$

$$L.H.S = \left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix} - \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$L.H.S = \left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\0-2 & -2+2 & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}-2-1 & 1-2 & -1-3\\-2-2 & 0-3 & 1-1\\-2-1 & 0+1 & -1-0\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix}$$

$R.H.S = (C - A) - B$

$$R.H.S = \left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix} - \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$R.H.S = \left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\0-2 & -2+2 & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}-2-1 & 1-2 & -1-3\\-2-2 & 0-3 & 1-1\\-2-1 & 0+1 & -1-1\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(viii). $(\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C} = \boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C})$

Solution.

$L.H.S = (A + B) + C$

$$L.H.S = \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2-1 & 1+0 & 4+0\\4+0 & 1-2 & 3+6\\4+1 & 0+1 & 3+2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}1 & 1 & 4\\4 & -1 & 9\\5 & 1 & 5\end{bmatrix}$$

$R.H.S = A + (B + C)$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1-1 & -1+0 & 1+0\\2+0 & -2-2 & 2+3\\3+1 & 1+1 & 3+2\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}0 & -1 & 1\\2 & -4 & 5\\4 & 2 & 5\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}1+0 & 2-1 & 3+1\\2+2 & 3-4 & 1+5\\1+4 & -1+2 & 0+5\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}1 & 1 & 4\\4 & -1 & 9\\5 & 1 & 5\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(ix). $\boldsymbol{A} + (\boldsymbol{B} - \boldsymbol{C}) = (\boldsymbol{A} - \boldsymbol{C}) + \boldsymbol{B}$

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Solution.

$$L.H.S = A + (B - C)$$

$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1+1 & -1-0 & 1-0 \\ 2-0 & -2+2 & 2-3 \\ 3-1 & 1-1 & 3-2 \end{bmatrix} \right)$$

$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 1+2 & 2-1 & 3+1 \\ 2+2 & 3+0 & 1-1 \\ 1+2 & -1+0 & 0+1 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

$$R.H.S = (A - C) + B$$

$$R.H.S = \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$R.H.S = \left(\begin{bmatrix} 1+1 & 2-0 & 3-0 \\ 2-0 & 3+2 & 1-3 \\ 1-1 & -1-1 & 0-2 \end{bmatrix} \right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 2+1 & 2-1 & 3+1 \\ 2+2 & 5-2 & -2+2 \\ 0+3 & -2+1 & -2+3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

Hence Proved. $\boldsymbol{L.H.S = R.H.S.}$

(x). $\boldsymbol{2A + 2B = 2(A + B)}$

Solution.

$$L.H.S = 2A + 2B$$

$$L.H.S = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 2+2 & 4-2 & 6+2 \\ 4+4 & 6-4 & 2+4 \\ 2+6 & -2+2 & 0+6 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 4 & 6 \\ 7 & 0 & 6 \end{bmatrix}$$

$$R.H.S = 2(A + B)$$

$$L.H.S = 2\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$R.H.S = 2\left(\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1!+3 & -1+1 & 0+3 \end{bmatrix} \right)$$

$$R.H.S = 2\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 4 & 6 \\ 7 & 0 & 6 \end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

Question.6. If $\boldsymbol{A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}}$ ***and*** $\boldsymbol{B =}$ $\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$, *find*

(i). $3A - 2B$

Solution.

$$3A - 2B = 3\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3-0 & -6-14 \\ 9+6 & 12-16 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

Answer.

(ii). $\boldsymbol{2A^t - 3B^t}$

Solution.

$$2A^t - 3B^t = 2\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}^t - 3\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}^t$$

$$= 2\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} - 3\begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 6+9 \\ -4-21 & 8-24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

Answer.

Question.7. If $\boldsymbol{2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} =}$ $\boldsymbol{\begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}}$, ***find***

Solution.

Given that

$$2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4+3 & 8+3b \\ -6+24 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

By the definition of the equal matrix, we have

$$8 + 3b = 10\ , \quad 2a - 12 = 1$$

$$3b = 10 - 8\ , \quad 2a = 1 + 12$$

$$b = \frac{2}{3}\ , \quad a = \frac{13}{2}$$

Hence $a = \frac{13}{2}$ *and* $b = \frac{2}{3}$.

Question.8. If $\boldsymbol{A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B =}$ $\boldsymbol{\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}}$, ***then verify that***

(i). $\boldsymbol{(A + B)^t = A^t + B^t}$

Solution.

$$L.H.S = (A + B)^t$$

$$L.H.S = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \right)^t$$

$$L.H.S=\left(\begin{bmatrix}1+1 & 2+1\\0+2 & 1+0\end{bmatrix}\right)^t$$
$$L.H.S=\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}^t$$
$$L.H.S=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$
$$R.H.S=A^t+B^t$$
$$R.H.S=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t+\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$R.H.S=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}1+1 & 0+2\\2+1 & 1+0\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

Hence Proved. $L.H.S=R.H.S.$

(ii). $(\boldsymbol{A}-\boldsymbol{B})^t=\boldsymbol{A}^t-\boldsymbol{B}^t$

Solution.

$$L.H.S=(A-B)^t$$
$$L.H.S=\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$$
$$L.H.S=\left(\begin{bmatrix}1-1 & 2-1\\0-2 & 1-0\end{bmatrix}\right)^t$$
$$L.H.S=\begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}^t$$
$$L.H.S=\begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$
$$R.H.S=A^t-B^t$$
$$R.H.S=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t-\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$R.H.S=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}-\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}1-1 & 0-2\\2-1 & 1-0\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$

Hence Proved. $L.H.S=R.H.S.$

(iii). $\boldsymbol{A}+\boldsymbol{A}^t$ ***is symmetric.***

Solution.

$$A+A^t=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$
$$A+A^t=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$
$$A+A^t=\begin{bmatrix}1+1 & 2+0\\0+2 & 1+1\end{bmatrix}$$
$$A+A^t=\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}---(i)$$

Now

$$(A+A^t)^t=\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}^t$$
$$(A+A^t)^t=\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

Using equation (i), we have

$$(A+A^t)^t=A+A^t$$

Hence $A+A^t$ *is symmetric.*

(iv). $\boldsymbol{A}-\boldsymbol{A}^t$ ***is Skew − symmetric.***

Solution.

$$A-A^t=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$
$$A-A^t=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$
$$A-A^t=\begin{bmatrix}1-1 & 2-0\\0-2 & 1-1\end{bmatrix}$$
$$A-A^t=\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}---(i)$$

Now

$$(A-A^t)^t=\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}^t$$
$$(A-A^t)^t=\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$$
$$(A-A^t)^t=-\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

Using equation (i), we have

$$(A-A^t)^t=-(A-A^t)$$

Hence $A-A^t$ *is Skew − symmetric.*

(iii). $\boldsymbol{B}+\boldsymbol{B}^t$ ***is symmetric.***

Solution.

$$B+B^t=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$B+B^t=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}+\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$B+B^t=\begin{bmatrix}1+1 & 1+2\\2+1 & 0+0\end{bmatrix}$$
$$B+B^t=\begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}---(i)$$

Now

$$(B+B^t)^t=\begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}^t$$
$$(B+B^t)^t=\begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}$$

Using equation (i), we have

$$(B+B^t)^t=B+B^t$$

Hence $B+B^t$ *is symmetric.*

(iii). $\boldsymbol{B}-\boldsymbol{B}^t$ ***is Skew − symmetric.***

Solution.

$$B-B^t=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$B-B^t=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}-\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$B-B^t=\begin{bmatrix}1-1 & 1-2\\2-1 & 0-0\end{bmatrix}$$
$$B-B^t=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}---(i)$$

Now

$$(B-B^t)^t=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}^t$$
$$(B-B^t)^t=\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$
$$(B-B^t)^t=-\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

Using equation (i), we have

$$(B-B^t)^t=-(B-B^t)$$

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Multiplication of Matrices:
Two matrices A and B are conformable for multiplication if
No of col of A =No. Of Rows of B

Exercise1.4

Q#1) Which of the following product matrices is conformable for multiplication?.

(i). $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

(ii). $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

(iii). $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Sol:
Not conformable for multiplication because
No of col of 1st Matrix= 1 ≠ 2 =No. Of Rows of 2nd Matrix

(iv). $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

(v). $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 3 =No. Of Rows of 2nd Matrix

Q#2) If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

Find (i). AB
(ii). BA **(if possible)**

(i). AB

Sol:

$$AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$$
$$= \begin{bmatrix} (3)(6) + (0)(5) \\ (-1)(6) + (2)(5) \end{bmatrix}$$
$$= \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix}$$
$$= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii). BA **(if possible)**

Sol:

$$BA = \begin{bmatrix} 6 \\ 5 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$$

Since
No of col of $A = 1 \neq 2$ =No. Of Rows of B
Multiplication is not possible.

Q#3) Find the following products.

(i). $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$
$$= [(1)(4) + (2)(0)]$$
$$= [4 + 0]$$
$$= [4]$$

(ii). $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Sol: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$
$$= [(1)(5) + (2)(-4)]$$
$$= [5 - 8]$$
$$= [-3]$$

(iii). $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol: $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$
$$= [(-3)(4) + (0)(0)]$$
$$= [-12 + 0]$$
$$= [-12]$$

(iv). $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol: $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$
$$= [(6)(4) + (0)(0)]$$
$$= [24 + 0]$$
$$= [24]$$

(v). $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Sol: $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$
$$= \begin{bmatrix} (1)(4) + (2)(0) & (1)(5) + (2)(-4) \\ (-3)(4) + (0)(0) & (-3)(5) + (0)(-4) \\ (6)(4) + (-1)(0) & (6)(5) + (-1)(-4) \end{bmatrix}$$
$$= \begin{bmatrix} 4 + 0 & 5 - 8 \\ -12 + 0 & -15 + 0 \\ 24 + 0 & 30 + 4 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$$

Q#4) Multiply the following matrices.

(a). $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

Sol: $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

$= \begin{bmatrix}(2)(2)+(3)(3) & (2)(-1)+(3)(0)\\(1)(2)+(1)(3) & (1)(-1)+(1)(0)\\(0)(2)+(-2)(3) & (0)(-1)+(-2)(0)\end{bmatrix}$

$= \begin{bmatrix}4+9 & -2+0\\2+3 & -1+0\\0-6 & 0+0\end{bmatrix}$

$= \begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$

(b). $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

Sol: $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

$= \begin{bmatrix}(1)(1)+(2)(3)+(3)(-1) & (1)(2)+(2)(4)+(3)(1)\\(4)(1)+(5)(3)+(6)(-1) & (4)(2)+(5)(4)+(6)(1)\end{bmatrix}$

$= \begin{bmatrix}1+6-3 & 2+8+3\\4+15-6 & 8+20+6\end{bmatrix}$

$= \begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$

(c). $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

Sol: $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

$= \begin{bmatrix}(1)(1)+(2)(4) & (1)(2)+(2)(5) & (1)(3)+(2)(6)\\(3)(1)+(4)(4) & (3)(2)+(4)(5) & (3)(3)+(4)(6)\\(-1)(1)+(1)(4) & (-1)(2)+(1)(5) & (-1)(3)+(1)(6)\end{bmatrix}$

$= \begin{bmatrix}1+8 & 2+10 & 3+12\\3+16 & 6+20 & 9+24\\-1+4 & -2+5 & -3+6\end{bmatrix}$

$= \begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$

(d). $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

Sol: $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

$=$

$\begin{bmatrix}(8)(2)+(5)(-4) & (8)\left(-\frac{5}{2}\right)+(5)(4)\\(6)(2)+(4)(-4) & (6)\left(-\frac{5}{2}\right)+(4)(4)\end{bmatrix}$

$= \begin{bmatrix}16-20 & -20+20\\12-16 & -15+16\end{bmatrix}$

$= \begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$

(e). $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Sol: $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$= \begin{bmatrix}(-1)(0)+(2)(0) & (-1)(0)+(2)(0)\\(1)(0)+(1)(0) & (1)(0)+(1)(0)\end{bmatrix}$

$= \begin{bmatrix}0+0 & 0+0\\0+0 & 0+0\end{bmatrix}$

$= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Q#5) Let $A = \begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$, $B = \begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$ **and** $C = \begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$, **Verify that**

(i). $AB = BA$

Sol: $L.H.S = AB$

$= \begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$

$= \begin{bmatrix}(-1)(1)+(3)(-3) & (-1)(2)+(3)(-5)\\(2)(1)+(0)(-3) & (2)(2)+(0)(-5)\end{bmatrix}$

$= \begin{bmatrix}-1-9 & -2-15\\2+0 & 4+0\end{bmatrix}$

$= \begin{bmatrix}-10 & -17\\2 & 4\end{bmatrix} \rightarrow (1)$

$: R.H.S = BA$

$= \begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$

$= \begin{bmatrix}(1)(-1)+(2)(2) & (1)(3)+(2)(0)\\(-3)(-1)+(-5)(2) & (-3)(3)+(-5)(0)\end{bmatrix}$

$= \begin{bmatrix}-1+4 & 3+0\\3-10 & -6+0\end{bmatrix}$

$= \begin{bmatrix}3 & 3\\-7 & -6\end{bmatrix} \rightarrow (2)$

From (1) and (2), we have

$AB \neq BA$

(ii). $A(BC) = (AB)C$

Sol:

$L.H.S = A(BC)$

$= \begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}\right)$

$= \begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\left(\begin{bmatrix}(1)(2)+(2)(1) & (1)(1)+(2)(3)\\(-3)(2)+(-5)(1) & (-3)(1)+(-5)(3)\end{bmatrix}\right)$

$= \begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}2+2 & 1+6\\-6-5 & -3-15\end{bmatrix}$

$= \begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\-11 & -18\end{bmatrix}$

$= \begin{bmatrix}(-1)(4)+(3)(-11) & (-1)(7)+(3)(-18)\\(2)(4)+(0)(-11) & (2)(7)+(0)(-18)\end{bmatrix}$

$= \begin{bmatrix}-4-33 & -7-54\\8+0 & 14+0\end{bmatrix}$

$= \begin{bmatrix}-37 & -61\\8 & 14\end{bmatrix} \rightarrow (1)$

$R.H.S = (AB)C$

$= \left(\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\right)\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$= \left(\begin{bmatrix}(-1)(1)+(3)(-3) & (-1)(2)+(3)(-5)\\(2)(1)+(0)(-3) & (2)(2)+(0)(-5)\end{bmatrix}\right)\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$= \begin{bmatrix}-1-9 & -2-15\\2+0 & 4+0\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$= \begin{bmatrix}-10 & -17\\2 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$= \begin{bmatrix}(-10)(2)+(-17)(1) & (-10)(1)+(-17)(3)\\(2)(2)+(4)(1) & (2)(1)+(4)(3)\end{bmatrix}$

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$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$

$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \ldots(2)$

From (1) and (2), we have

$A(BC) = (AB)C$

(iii). $A(B+C) = AB + AC$

Sol:

$L.H.S = A(BC)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$

$= \begin{bmatrix} (-1)(3)+(3)(-2) & (-1)(3)+(3)(-2) \\ (2)(3)+(0)(-2) & (2)(3)+(0)(-2) \end{bmatrix}$

$= \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$

$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix} \to (1)$

$R.H.S = AB + BC$

$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right) + \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$

$+ \begin{bmatrix} (-1)(2)+(3)(1) & (-1)(1)+(3)(3) \\ (2)(2)+(0)(1) & (2)(1)+(0)(3) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} + \begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -10+1 & -17+8 \\ 2+4 & 4+2 \end{bmatrix}$

$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix} \to (2)$

From (1) and (2), we have

$A(B+C) = AB + AC$

(iv). $A(B-C) = AB - AC$

Sol:

$L.H.S = A(BC)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1-2 & 2-1 \\ -3-1 & -5-3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -4 & -8 \end{bmatrix}$

$= \begin{bmatrix} (-1)(-1)+(3)(-4) & (-1)(1)+(3)(-8) \\ (2)(-1)+(0)(-4) & (2)(1)+(0)(-8) \end{bmatrix}$

$= \begin{bmatrix} 1-12 & -1-24 \\ -2+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix} \ldots(1)$

$R.H.S = AB - BC$

$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right) - \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$

$- \begin{bmatrix} (-1)(2)+(3)(1) & (-1)(1)+(3)(3) \\ (2)(2)+(0)(1) & (2)(1)+(0)(3) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} - \begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -10-1 & -17-8 \\ 2-4 & 4-2 \end{bmatrix}$

$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix} \to (2)$

From (1) and (2), we have

$A(B-C) = AB - AC$

Q#6) For the matrices $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$, verify that

(i). $(AB)^t = B^tA^t$

Sol: : $L.H.S = (AB)^t$

First we find AB

$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$

$\text{AB} = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$

Taking transpose on both side

$(\text{AB})^t = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}^t$

$(\text{AB})^t = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix} \ldots(1)$

$: R.H.S = B^tA^t$

$= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}^t$

$= \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} (1)(-1)+(-3)(3) & (1)(2)+(-3)(0) \\ (2)(-1)+(-5)(3) & (2)(2)+(-5)(0) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & 2+0 \\ -2-15 & 4+0 \end{bmatrix}$

$B^tA^t = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix} \to (2)$

From (1) and (2), we have

$(AB)^t = B^tA^t$

(ii). $(BC)^t = C^tB^t$

Sol: $L.H.S = (BC)^t$

First we find BC

$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

$= \begin{bmatrix} (1)(-2)+(2)(3) & (1)(6)+(2)(-9) \\ (-3)(-2)+(-5)(3) & (-3)(6)+(-5)(-9) \end{bmatrix}$

$= \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix}$

$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix} \to (2)$

Taking transpose on both side

$(AB)^t = \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$

$(AB)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \rightarrow (1)$

$: R.H.S = C^t B^t$

$= \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}^t \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$

$= \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$

$= \begin{bmatrix} (-2)(1) + (3)(2) & (-2)(-3) + (3)(-5) \\ (6)(1) + (-9)(2) & (6)(-3) + (-9)(-5) \end{bmatrix}$

$= \begin{bmatrix} -2 + 6 & 6 - 15 \\ 6 - 18 & -18 + 45 \end{bmatrix}$

$C^t B^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \rightarrow (2)$

From (1) and (2), we have

$$(BC)^t = C^t B^t$$

Determinant of 2×2 matrix:

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2×2 square matrix, the determinant of A is denoted by $|A|$ or detA

And given as

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$= (a)(d) - (b)(c)$

$= ad - bc$

For example, $\boldsymbol{A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$= (-1)(0) - (1)(2)$

$= 0 - 2 = -2$

Singular and Non-singular matrices:

Singular matrix:

A square matrix A is called Singular matrix if its determinant is zero i.e.$|A| = 0$

For example, $A = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$

$= (3)(2) - (3)(2)$

$= 6 - 6 = 0$

Non-Singular matrix:

A square matrix A is called Non-Singular matrix if its determinant is not zero i.e.$|A| \neq 0$

For example, $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$= (-1)(0) - (1)(2)$

$= 0 - 2 = -2 \neq 0$

Adjoint of Matrix A:

"Adjoint of a square matrix $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries."

For example, $A = \begin{bmatrix} -1 & 4 \\ 2 & 0 \end{bmatrix}$

$AdjA = \begin{bmatrix} 0 & -4 \\ -2 & -1 \end{bmatrix}$

Exercise 1.5

Q#1) Find the determinant of the following matrices.

(i). $\boldsymbol{A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}}$

Sol:

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$= (-1)(0) - (1)(2)$

$= 0 - 2 = -2$

(ii). $\boldsymbol{B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}}$

Sol:

$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$

$= (1)(-2) - (3)(2)$

$= -2 - 6 = -8$

(iii). $\boldsymbol{C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}}$

Sol:

$|C| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$

$= (3)(2) - (3)(2)$

$= 6 - 6 = 0$

(iv). $\boldsymbol{D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}}$

Sol:

$|D| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$

$= (3)(4) - (2)(1)$

$= 12 - 2 = 10$

Q#2)

Find which of the following matrices are singular or non-singular?

(i). $\boldsymbol{A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}}$

Sol:

$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$

$= (3)(4) - (6)(2)$

$= 12 - 12 = 0$

Hence, matrix A is singular matrix.

(ii). $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Sol:

$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$

$= (4)(2) - (1)(3)$

$= 8 - 3 = 5$

Which is not zero and hence, matrix A is Non-singular matrix.

(iii). $\boldsymbol{C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}}$

Sol:

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$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$
$= (7)(5) - (-9)(3)$
$= 35 + 27 = 62$
Which is not zero and hence, matrix A is Non-singular matrix.

(iv). $\boldsymbol{D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}}$

Sol:
$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$
$= (5)(4) - (-10)(-2)$
$= 20 - 20 = 0$
Hence, matrix A is singular matrix.

Q#3) Find the multiplicative inverse (if exists) of each:

(i). $\boldsymbol{A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}}$

Sol: First we find the determinant of A as
$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$
$= (-1)(0) - (3)(2)$
$= 0 - 6 = -6$
Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

As
$$A^{-1} = \frac{1}{|A|} AdjA$$
Putting values
$$A^{-1} = \frac{1}{-6}\begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} \frac{0}{-6} & \frac{-3}{-6} \\ \frac{-2}{-6} & \frac{-1}{-6} \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii). $\boldsymbol{B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}}$

Sol: First we find the determinant of B as
$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$
$= (1)(-5) - (2)(-3)$
$= -5 + 6 = 1$
Which is not zero and hence, matrix B is Non-singular matrix and B^{-1} exist.

Now, $AdjB = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

As
$$B^{-1} = \frac{1}{|B|} AdjB$$
Putting values
$$B^{-1} = \frac{1}{1}\begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

(iii). $\boldsymbol{C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}}$

Sol: First we find the determinant of C as
$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$
$= (-2)(-9) - (3)(6)$
$= 18 - 18 = 0$
Which is zero and hence, matrix C is singular matrix and C^{-1} does not exist.

(iv). $\boldsymbol{D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}}$

Sol: First we find the determinant of D as
$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$
$= \left(\frac{1}{2}\right)(2) - \left(\frac{3}{4}\right)(1) = 1 - \frac{3}{4}$
$= \frac{4-3}{4} = \frac{1}{4}$
Which is not zero and hence, matrix D is Non-singular matrix and D^{-1} exist.

Now, $AdjD = \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$

As
$$D^{-1} = \frac{1}{|\mathrm{D}|} AdjD$$
Putting values
$$D^{-1} = \frac{1}{\frac{1}{4}}\begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} = \frac{4}{1}\begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$
$$D^{-1} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q#4) If $\boldsymbol{A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}}$ and $\boldsymbol{B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}}$, then

(i). $\boldsymbol{A(AdjA) = (AdjA)A = (detA)I}$

Sol: First we find the determinant of A as
$|A| = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix}$
$= (1)(6) - (2)(4)$
$= 6 - 8 = -2$

Now, $AdjA = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

Let $A(AdjA) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$
$= \begin{bmatrix} (1)(6) + (2)(-4) & (1)(-2) + (2)(1) \\ (4)(6) + (6)(-4) & (4)(-2) + (6)(1) \end{bmatrix}$
$= \begin{bmatrix} 6-8 & -2+2 \\ 24-24 & -8+6 \end{bmatrix}$
$A(AdjA) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ ...(1)

And $(AdjA)A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$
$= \begin{bmatrix} (6)(1) + (-2)(4) & (6)(2) + (-2)(6) \\ (-4)(1) + (1)(4) & (-4)(2) + (1)(6) \end{bmatrix}$

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$= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$

$(AdjA)A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$...(2)

Also, $(detA)I = -2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$...(3)

From Eq(1), (2) and (3), we have

$A(AdjA) = (AdjA)A = (detA)I$

(ii). $BB^{-1} = B^{-1}B = I$

Sol: First we find the determinant of

$|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$

$= (3)(-2) - (-1)(2)$

$= -6 + 2 = -4$

Now, $AdjB = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

As

$B^{-1} = \frac{1}{|B|}AdjB$

Putting values

$B^{-1} = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

Let $BB^{-1} = \frac{1}{-4}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} (3)(-2)+(-1)(-2) & (3)(1)+(-1)(3) \\ (2)(-2)+(-2)(-2) & (2)(1)+(-2)(3) \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -6+2 & 3-3 \\ -4+4 & 2-6 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \rightarrow$(1)

Also $B^{-1}B = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} (-2)(3)+(1)(2) & (-2)(-1)+(1)(-2) \\ (-2)(3)+(3)(2) & (-2)(-1)+(3)(-2) \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -6+2 & 2-2 \\ -6+6 & 2-6 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \rightarrow$(2)

From (1) and (2), we have

$BB^{-1} = B^{-1}B = I.$

Q#5) Determine whether the given matrices are multiplicative inverse of each other or not.

(i). $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

Sol:

$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} (3)(7)+(5)(-4) & (3)(-5)+(5)(3) \\ (4)(7)+(7)(-4) & (4)(-5)+(7)(3) \end{bmatrix}$

$= \begin{bmatrix} 21-20 & -15+15 \\ 28-28 & -20+21 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Yes, the given matrices are multiplicative inverse of each other.

(ii). $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Sol:

$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} (1)(-3)+(2)(2) & (1)(2)+(2)(-1) \\ (2)(-3)+(3)(2) & (2)(2)+(3)(-1) \end{bmatrix}$

$= \begin{bmatrix} -3+4 & 2-2 \\ -6+6 & 4-3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Yes, the given matrices are multiplicative inverse of each other.

Q#6) If $A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$, then verify that

(i). $(AB)^{-1} = B^{-1}A^{-1}$

Sol: L.H.S=$(AB)^{-1}$

First we find

$AB = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} (4)(-4)+(0)(1) & (4)(-2)+(0)(-1) \\ (-1)(-4)+(2)(1) & (-1)(-2)+(2)(-1) \end{bmatrix}$

$= \begin{bmatrix} -16+0 & -8+0 \\ 4+2 & 2-2 \end{bmatrix}$

$= \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$

Now, we find the its determinant

$|AB| = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$

$= (-16)(0) - (-8)(6)$

$= 0 - (-48) = 48$

Which is not zero and hence, matrix AB is Non-singular matrix and $(AB)^{-1}$ exist.

Now, $AdjAB = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$

As

$(AB)^{-1} = \frac{1}{|AB|}AdjAB$

Putting values

$L.H.S = (AB)^{-1} = \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$...(1)

$R.H.S = B^{-1}A^{-1}$

First, we find B^{-1} and A^{-1}

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$= (4)(2) - (0)(-1)$

$= 8 - 0 = 8$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}AdjA$

Putting values

$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

Also, $|B| = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$

$= (-4)(-1) - (-2)(1)$

$= 4 + 2 = 6$
Which is not zero and hence, matrix B is Non-singular matrix and B^{-1} exist.

Now, $AdjB = \begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} AdjB$

Putting values

$B^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix}$

$R.H.S = B^{-1}\,A^{-1}$

$= \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix}\frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$= \frac{1}{8\times 6}\begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} (-1)(2) + (2)(1) & (-1)(0) + (2)(4) \\ (-1)(2) + (-4)(1) & (-1)(0) + (-4)(4) \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} -2 + 2 & 0 + 8 \\ -2 - 4 & 0 - 16 \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$…(2)

From (1) and (2), we have

$\boldsymbol{(AB)^{-1} = B^{-1}\,A^{-1}}$

(ii). $\boldsymbol{(DA)^{-1} = A^{-1}D^{-1}}$

Sol: L.H.S=$\boldsymbol{(DA)^{-1}}$

First we find

$DA = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (3)(4) + (1)(-1) & (1)(0) + (1)(2) \\ (-2)(4) + (2)(-1) & (-2)(0) + (2)(2) \end{bmatrix}$

$= \begin{bmatrix} 12 - 1 & 0 + 2 \\ -8 - 2 & 0 - 4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$

Now, we find the its determinant

$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$

$= (11)(4) - (2)(-10)$

$= 44 + 20 = 64$

Which is not zero and hence, matrix DA is Non-singular matrix and $(DA)^{-1}$ exist.

Now, $AdjDA = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$

$(DA)^{-1} = \frac{1}{|DA|} AdjDA$

Putting values

$L.H.S = (DA)^{-1} = \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$…(1)

$R.H.S = A^{-1}D^{-1}$

First, we find D^{-1} and A^{-1}

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$= (4)(2) - (0)(-1)$

$= 8 - 0 = 8$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} AdjA$

Putting values

$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

Also, $|D| = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}$

$= (3)(2) - (1)(-2)$

$= 6 + 2 = 8$

Which is not zero and hence, matrix D is Non-singular matrix and D^{-1} exist.

Now, $AdjD = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$D^{-1} = \frac{1}{|D|} AdjD$

Putting values

$D^{-1} = \frac{1}{8}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$R.H.S = A^{-1}D^{-1}$

$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\frac{1}{8}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{8\times 8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} (2)(2) + (0)(-2) & (2)(-1) + (0)(3) \\ (1)(2) + (4)(-2) & (1)(-1) + (4)(3) \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} 4 + 0 & -2 + 0 \\ 2 + 8 & -1 + 12 \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$…(2)

From (1) and (2), we have

$(DA)^{-1} = A^{-1}\,D^{-1}$

Exercise 1.6

Q#1) Use matrices, to solve the following system of linear equations by:
(a). the matrix inverse method
(b). the Cramer's rule
(i). $\mathbf{2x - 2y = 4}$; $\mathbf{3x + 2y = 6}$

Sol: (a). the matrix inverse method
In matrix form
$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$
$AX = B \Rightarrow X = A^{-1}B \rightarrow$(1)
Where $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$
Now, we find A^{-1} using
$A^{-1} = \frac{1}{|A|} AdjA \rightarrow$(2)
$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$
$= (2)(2) - (-2)(3)$
$= 4 + 6 = 10$
Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.
Now, $AdjA = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$
Putting values in eq. (2), we have
$A^{-1} = \frac{1}{|A|} AdjA$
$\Rightarrow A^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$
Now, put values in eq. (1)
$X = A^{-1}B$
$\Rightarrow X = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$
$= \frac{1}{10}\begin{bmatrix} (2)(4) + (2)(6) \\ (-3)(4) + (2)(6) \end{bmatrix}$
$= \frac{1}{10}\begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix}$
$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$
$\Rightarrow \quad x = 2, y = 0$

(b). the Cramer's rule
In matrix form
$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$
Where $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$ and
$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$
First of all we find $|A|, |A_x| and\ |A_y|$
$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$
$\Rightarrow |A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$
$= (2)(2) - (-2)(3)$
$|A| = 4 + 6 = 10$
Which is non-zero, so solution exists and
$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$
$\Rightarrow |A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}$
$|A_x| = (4)(2) - (-2)(6)$
$|A_x| = 8 + 12 = 20$
Also,
$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$
$\Rightarrow |A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$
$= (2)(6) - (4)(3)$
$|A_y| = 12 - 12 = 0$
Now
$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{20}{10} = 2$
And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{0}{10} = 0$
Hence, $x = 2$ and $y = 0$

(ii). $\mathbf{2x + y = 3}$; $\mathbf{6x + 5y = 1}$

Sol: (a). the matrix inverse method
In matrix form
$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$AX = B \Rightarrow X = A^{-1}B$...(1)
Where $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$
Now, we find A^{-1} using
$A^{-1} = \frac{1}{|A|} AdjA \rightarrow$(2)
$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$
$= (2)(5) - (1)(6)$
$= 10 - 6 = 4$
Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.
Now, $AdjA = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$
Putting values in eq. (2), we have
$A^{-1} = \frac{1}{|A|} AdjA$
$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$
Now, put values in eq. (1)
$X = A^{-1}B$
$\Rightarrow X = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$
$= \frac{1}{4}\begin{bmatrix} (5)(3) + (-1)(1) \\ (-6)(3) + (2)(1) \end{bmatrix}$
$= \frac{1}{4}\begin{bmatrix} 15 - 1 \\ -18 + 2 \end{bmatrix}$
$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix} = \begin{bmatrix} \frac{14}{4} \\ -\frac{16}{4} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$

$\Rightarrow \quad x = \frac{7}{2}, y = -4$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Where $A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$, $A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$ and

$$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$

$= (2)(5) - (1)(6)$

$= 10 - 6 = 4$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}$

$|A_x| = (3)(5) - (1)(1)$

$|A_x| = 15 - 1 = 14$

Also,

$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}$

$= (2)(1) - (3)(6)$

$|A_y| = 2 - 18 = -16$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{14}{4} = \frac{7}{2}$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{-16}{4} = -4$

Hence, $x = \frac{7}{2}$ and $y = -4$

(iii). $4x + 2y = 8$; $3x - y = -1$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} AdjA$...(2)

$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (2)(3)$

$= -4 - 6 = -10$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|} AdjA$

$\Rightarrow A^{-1} = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} (-1)(8) + (-2)(-1) \\ (-3)(8) + (4)(-1) \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} -8 + 2 \\ -24 - 4 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} -6 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{-6}{-10} \\ \frac{-28}{-10} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{7}{5} \end{bmatrix}$

$\Rightarrow \quad x = \frac{3}{5}, y = \frac{7}{5}$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Where $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$ and $A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (2)(3)$

$= -4 - 6 = -10$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$

$|A_x| = (8)(-1) - (2)(-1)$

$|A_x| = -8 + 2 = -6$

Also,

$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (8)(3)$

$|A_y| = -4 - 24 = -28$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-6}{-10} = \frac{3}{5}$

And $y=\frac{|A_y|}{|A|} \Rightarrow \mathrm{y}=\frac{-28}{-10}=\frac{7}{5}$

Hence, $x=\frac{3}{5}$ and $y=\frac{7}{5}$

(iv). $\boldsymbol{3x-2y=-6}$; $\boldsymbol{5x-2y=-10}$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

$AX=B \Rightarrow X=A^{-1}B \rightarrow$(1)

Where $A=\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$ and $B=\begin{bmatrix}-6\\-10\end{bmatrix}$

Now, we find A^{-1} using

$A^{-1}=\frac{1}{|A|}AdjA \rightarrow$(2)

$|A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$

$=(3)(-2)-(-2)(5)$

$=-6+10=4$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA=\begin{bmatrix}-2 & 2\\-5 & 3\end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1}=\frac{1}{|A|}AdjA$

$\Rightarrow A^{-1}=\frac{1}{4}\begin{bmatrix}-2 & 2\\-5 & 3\end{bmatrix}$

Now, put values in eq. (1)

$X=A^{-1}B$

$\Rightarrow \mathrm{X}=\frac{1}{4}\begin{bmatrix}-2 & 2\\-5 & 3\end{bmatrix}\begin{bmatrix}-6\\-10\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}(-2)(-6)+(2)(-10)\\(-5)(-6)+(3)(-10)\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}12-20\\30-30\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}-8\\0\end{bmatrix}=\begin{bmatrix}\frac{-8}{4}\\\frac{0}{4}\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\0\end{bmatrix}$

$\Rightarrow \quad \mathrm{x}=-2, \mathrm{y}=0$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

Where $A=\begin{bmatrix}3 & 2\\5 & -2\end{bmatrix}$, $A_x=\begin{bmatrix}-6 & 2\\-10 & -2\end{bmatrix}$ and

$A_y=\begin{bmatrix}3 & -6\\5 & -10\end{bmatrix}$

First of all we find $|A|, |A_x| and\ |A_y|$

$A=\begin{bmatrix}3 & 2\\5 & -2\end{bmatrix}$

$\Rightarrow |A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$

$=(3)(-2)-(-2)(5)$

$=-6+10=4$

Which is non-zero, so solution exists and

$A_x=\begin{bmatrix}-6 & 2\\-10 & -2\end{bmatrix}$

$\Rightarrow |A_x|=\begin{vmatrix}-6 & 2\\-10 & -2\end{vmatrix}$

$|A_x|=(-6)(-2)-(2)(-10)$

$|A_x|=12-20=-8$

Also,

$A_y=\begin{bmatrix}3 & -6\\5 & -10\end{bmatrix}$

$\Rightarrow |A_y|=\begin{vmatrix}3 & -6\\5 & -10\end{vmatrix}$

$=(3)(-10)-(-6)(5)$

$|A_y|=-30+30=0$

Now

$x=\frac{|A_x|}{|A|} \Rightarrow \mathrm{x}=\frac{-8}{4}=-2$

And $y=\frac{|A_y|}{|A|} \Rightarrow \mathrm{y}=\frac{0}{4}=0$

Hence, $x=-2$ and $y=0$

(iii). $\boldsymbol{3x-2y=4}$; $\boldsymbol{-6x+4y=7}$

Sol: The matrix inverse method

In matrix form

$$\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\7\end{bmatrix}$$

$AX=B \Rightarrow X=A^{-1}B \rightarrow$(1)

Where $A=\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$ and $B=\begin{bmatrix}4\\7\end{bmatrix}$

Now, we find A^{-1} using

$A^{-1}=\frac{1}{|A|}AdjA \rightarrow$(2)

$|A|=\begin{vmatrix}3 & -2\\-6 & 4\end{vmatrix}$

$=(3)(4)-(-2)(-6)$

$=12-12=0$

Which is zero and hence, matrix A is singular matrix and A^{-1} does not exist. No solution possible.

(vi). $\boldsymbol{4x+y=9}$; $\boldsymbol{-3x-y=-5}$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}9\\-5\end{bmatrix}$$

$AX=B \Rightarrow X=A^{-1}B \rightarrow$(1)

Where $A=\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$ and $B=\begin{bmatrix}9\\-5\end{bmatrix}$

Now, we find A^{-1} using

$A^{-1}=\frac{1}{|A|}AdjA \rightarrow$(2)

$|A|=\begin{vmatrix}4 & 1\\-3 & -1\end{vmatrix}$

$=(4)(-1)-(-3)(1)$

$=-4+3=-1$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA=\begin{bmatrix}-1 & -1\\3 & 4\end{bmatrix}$

Putting values in eq (2), we have

$A^{-1}=\frac{1}{|A|}AdjA$

$\Rightarrow A^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$= \frac{1}{-1}\begin{bmatrix} (-1)(9) + (-1)(-5) \\ (3)(9) + (4)(-5) \end{bmatrix}$

$= \frac{1}{-1}\begin{bmatrix} -9 + 5 \\ 27 - 20 \end{bmatrix}$

$= \frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{-4}{-1} \\ \frac{7}{-1} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$

$\Rightarrow \quad x = 4, y = -7$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Where $A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$ and

$A_y = \begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$

$= (4)(-1) - (-3)(1)$

$= -4 + 3 = -1$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}$

$|A_x| = (9)(-1) - (1)(-5)$

$|A_x| = -9 + 5 = -4$

Also,

$|A_y| = \begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix}$

$= (4)(-5) - (9)(-3)$

$A_y = -20 + 27 = 7$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-4}{-1} = 4$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{7}{-1} = 7$

Hence, $x = 4$ and $y = 7$

(vii). $\mathbf{2x - 2y = 4}$; $\mathbf{-5x - 2y = -10}$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} Adj A \rightarrow (2)$

$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$

$= (2)(-2) - (-2)(-5)$

$= -4 - 10 = -14$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $Adj A = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|} Adj A$

$\Rightarrow A^{-1} = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} (-2)(4) + (2)(-10) \\ (5)(4) + (2)(-10) \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} -8 - 20 \\ 20 - 20 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{-14} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\Rightarrow \quad x = 2, y = 0$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} \mathbf{2} & \mathbf{-2} \\ \mathbf{-5} & \mathbf{-2} \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \mathbf{4} \\ \mathbf{-10} \end{bmatrix}$$

Where $A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$, $A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$ and

$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$

$= (2)(-2) - (-2)(-5)$

$= -4 - 10 = -14$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix}$

$|A_x| = (4)(-2) - (-2)(-10)$

$|A_x| = -8 - 20 = -28$

Also,

$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix}$
$= (2)(-10) - (4)(-5)$
$|A_y| = -20 + 20 = 0$
Now
$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-28}{-14} = 2$
And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{0}{-14} = 0$
Hence, $x = 2$ and $y = 0$

(viii). $3x - 4y = 4$; $x + 2y = 8$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$
Where $A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$
Now, we find A^{-1} using
$A^{-1} = \frac{1}{|A|} AdjA$...(2)
$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$
$= (3)(2) - (-4)(1)$
$= 6 + 4 = 10$
Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.
Now, $AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$
Putting values in eq.(2), we have
$A^{-1} = \frac{1}{|A|} AdjA$
$\Rightarrow A^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$
Now, put values in eq. (1)
$X = A^{-1}B$
$\Rightarrow X = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$
$= \frac{1}{10}\begin{bmatrix} (2)(4) + (4)(8) \\ (-1)(4) + (3)(8) \end{bmatrix}$
$= \frac{1}{10}\begin{bmatrix} 8 + 32 \\ -4 + 24 \end{bmatrix}$
$= \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$
$\Rightarrow x = 4, y = 2$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Where $A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$, $A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$ and $A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$

First of all, we find $|A|, |A_x| and\ |A_y|$
$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$
$\Rightarrow |A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$
$= (3)(2) - (-4)(1)$
$= 6 + 4 = 10$
Which is non-zero, so solution exists and
$A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$
$\Rightarrow |A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$
$= (4)(2) - (-4)(8)$
$|A_x| = 8 + 32 = 40$
Also,
$A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$
$\Rightarrow |A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$
$= (3)(8) - (4)(1)$
$|A_y| = 24 - 4 = 20$
Now
$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{40}{10} = 4$
And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{20}{10} = 2$
Hence, $x = 4$ and $y = 2$

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MATHEMATICS 9th Science Group

7/18/2020

Chapter 2.

REAL AND COMPLEX NUMBERS

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Contents

Rational number:

A number which can be written in the form of $\frac{p}{q}$, $where\ p, q \in Z \wedge \neq 0$ is called a rational number.

e.g. $\frac{3}{4}, \frac{22}{7}, \frac{2}{6}$.

Irrational number:

A real number which cannot be written in the form of $\frac{p}{q}$, $where\ p, q \in Z \wedge \neq 0$ is called an irrational number.

e.g. $\sqrt{2}$, $\sqrt{5}$

Real number:

The field of all rational and irrational numbers is called the real numbers, or simply the "reals," and denoted $\mathbb{R}$.

Terminating decimal:

A decimal which has only a finite number of digits in its decimal part, is called terminating decimal. e.g. 202.04, 0.25, 0.5 example of terminating decimal.

Recurring decimal:

A decimal in which one or more digits repeats indefinitely is called recurring decimal or periodic decimal.

e.g. 0.33333 , 21.134134 … …

Exercise 2.1

Question.1. Identify which of the following are rational and irrational numbers

(i). $\sqrt{3}$

Solution.

Is an irrational number.

(ii). $\frac{1}{6}$

Solution.

Is a rational number.

(iii). π

Solution.

Is an irrational number.

(iv). $\frac{15}{7}$

Solution.

Is a rational number.

(v). 7.25

Solution.

Is a rational number.

(vi). $\sqrt{29}$

Solution.

Is an irrational number.

Question.2. Convert the following fractions into decimal fraction.

(i) $\frac{17}{25}$

Solution.

$$0.68$$

(ii) $\frac{19}{4}$

Solution.

$$4.75$$

(iii) $\frac{57}{8}$

Solution.

$$7.125$$

(iv) $\frac{205}{18}$

Solution.

$$11.3889$$

(v) $\frac{5}{8}$

Solution.

$$0.625$$

(vi) $\frac{25}{38}$

Solution.

$$0.65789$$

Question.3. Which of the following statements are true and which are false?

(i). $\frac{2}{3}$ *is an irrational number.*

Solution.

False.

(ii). π *is an irrational number.*

Solution.

True.

(iii). $\frac{1}{9}$ *is a terminating fraction.*

Solution.

False.

(iv). $\frac{3}{4}$ *is terminating fraction.*

Solution.

True.

(v). $\frac{4}{5}$ *is a recurring fraction..*

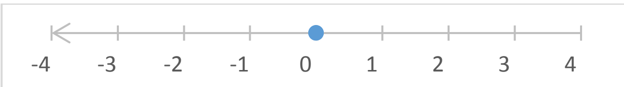
Solution.

False.

Question.4. Represent the following numbers on the number line

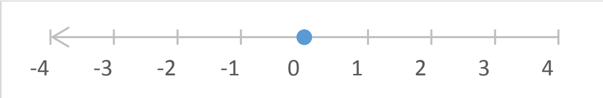
(i) $\frac{2}{3}$

Solution.



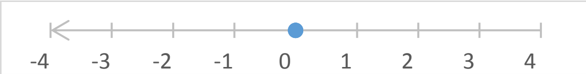
(ii). $-\frac{4}{5}$

Solution.



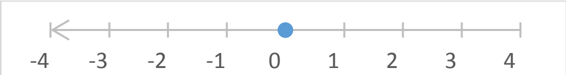
(iii). $1\frac{3}{4}$

Solution.



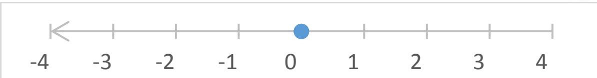
(iv). $-2\frac{5}{8}$

Solution.



(v). $\sqrt{5}$

Solution.



Question.5. Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Solution.

The mean of the numbers is between given numbers. Therefore

$$\text{required number is} = \frac{\frac{3}{4}+\frac{5}{9}}{2}$$

$$= \frac{\frac{27+20}{36}}{2}$$

$$= \frac{\frac{47}{36}}{2}$$

$$= \frac{47}{36\times 2}$$

$$= \frac{47}{72}$$

Answer.

Question.6. Express the following recurring decimals as the rational number $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

(i). $0.\overline{5}$

Solution.

Let

$$x = 0.\overline{5}$$

That is

$$x = 0.5555\ldots \rightarrow (i)$$

Only one digit 5 is being repeated, multiply by 10 on both sides of (i), we have

$$10x = (0.5555\ldots)\times 10$$

$$10x = 5.5555\ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$10x - x = 5.5555\ldots. - 0.5555\ldots$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$0.\overline{5} = \frac{5}{9}$$

Answer.

(ii). $0.\overline{13}$

Solution.

Let

$$x = 0.\overline{13}$$

That is

$$x = 0.13131313\ldots \rightarrow (i)$$

Only two digits 13 is being repeated, multiply by 100 on both sides of (i), we have

$$100x = (0.13131313\ldots)\times 100$$

$$100x = 13.13131313\ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$100x - x$$

$$= 13.13131313\ldots. - 0.13131313\ldots$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$0.\overline{13} = \frac{13}{99}$$

Answer.

(iii). $0.\overline{67}$

Solution.

Let

$$x = 0.\overline{67}$$

That is

$$x = 0.67676767\ldots \rightarrow (i)$$

Only two digits 67 is being repeated, multiply by 100 on both sides of (i), we have

$$100x = (0.67676767\ldots)\times 100$$

$$100x = 67.67676767\ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$100x - x$$

$$= 67.67676767\ldots. - 0.67676767\ldots$$

$$99x = 67$$

$$x = \frac{67}{99}$$

$$0.\overline{67} = \frac{67}{99}$$

Answer.

Properties of Real Numbers:

Binary Operations:

A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A to another element of A. e.g. two important binary operations are addition and multiplication in a set of real numbers.

Addition Laws:

Closure Law of Addition:

$$\forall\, a\,, b\ \in \Re\ then\ a+b \in \Re \qquad \forall\ stands\ for\ all.$$

Associative Law of Addition:

$$\forall\, a\,, b\,, c \in \Re\ then\ a+(b+c)=(a+b)+c.$$

Additive Identity:

$$\forall\, a \in \Re\,, \exists\, 0 \in \Re\ \ such\ that\ a+0=0+a=a.$$

$\exists$ *Stands for there exist and* 0 *is called the iditive identity.*

Additive Inverse:

$$\forall\, a \in \Re\,, \exists -a \in \Re\ \ such\ that\ a+(-a) = -a+a=0.$$

$-a$ *and* a *are called the idetive inverse of each other.*

Commutative Law for Addition:

$$\forall\, a\,, b\ \in \Re\ then\ a+b=b+a.$$

Multiplication Laws:

Closure Law of Multiplication:

$$\forall\, a\,, b\ \in \Re\ then\ ab \in \Re \qquad \forall\ stands\ for\ all.$$

Associative Law of Multiplication:

$$\forall\, a\,, b\,, c \in \Re\ then\ a(bc)=(ab)c.$$

Multiplicative Identity:

$$\forall\, a \in \Re\,, \exists\, 1 \in \Re\ \ such\ that\ a.1=1.a=a.$$

$\exists$ *Stands for there exist and* 1 *is called the iditive identity.*

Multiplicative Inverse:

$$\forall\, a \in \Re\,, \exists\, a' = \frac{1}{a} \in \Re\ \ such\ that\ a.\frac{1}{a} = \frac{1}{a}.a = 1.$$

a *and* $\frac{1}{a}$ *are called the idetive inverse of each other.*

Commutative Law for Multiplication:

$$\forall\, a\,, b\ \in \Re\ then\ ab=ba.$$

Properties of Equality:

Reflexive property:

$$\forall\, a\ \in \Re\ then\ a=a$$

Symmetric Property:

$$\forall\, a\,, b\ \in \Re\ and\ if\ a=b\ then\ b=a.$$

Transitive Property:

$$\forall\, a\,, b\,, c \in \Re\,, if\ a=b\ and\ b=c\ then\ a=c.$$

Additive Property:

$$\forall\, a\,, b\,, c \in \Re\,, a=b\ then\ a+c=b+c.$$

Multiplicative Property:

$$\forall\, a\,, b\,, c \in \Re\,, a=b\ then\ ac=bc.$$

Cancellation Property w.r.t. addition:

$$\forall\, a\,, b\,, c \in \Re\,, a+c=b+c\ then\ a=b.$$

Cancellation Property w.r.t. Multiplication:

$$\forall\, a\,, b\,, c \in \Re\,, ac=bc\ then\ a=b.$$

Distributive property of multiplication over addition.

$$a(b+c)=ab+ac$$

Distributive property of multiplication over Subtraction.

$$a(b-c)=ab-ac$$

Properties of Inequalities (Order properties):

Trichotomy Property:

$$\forall\, a\,, b\ \in \Re$$
$$either\ a=b\ or\ a>b\ or\ a<b.$$

Transitive Property:

$$\forall\, a\,, b\ \in \Re$$

(i). $if\ a>b\ and\ b>c\ then\ a>c.$

(ii). $if\ a<b\ and\ b<c\ then\ a<c.$

Additive Property:

$$\forall\, a\,, b\ \in \Re$$

(i). $if\ a>b\ then\ \ a+c>b+c.$

(ii). $if\ a<b\ and\ then\ a+c<b+c.$

Multiplicative Properties:

$$\forall\, a\,, b\,, c \in \Re$$

$If\ c>0$

(i). $if\ a > b\ then\ \ ac > bc.$
(ii). $if\ a < b\ and\ then\ ac < bc.$
$If\ c < 0$
(iii). $if\ a > b\ then\ \ ac < bc.$
(iv). $if\ a < b\ and\ then\ ac > bc.$

Exercise 2.2

Question.1. Identify the property used in the following.

(i). $\boldsymbol{a + b = b + a}$

Solution.

$Commutative\ property\ w.r.t\ Addition.$

(ii). $\boldsymbol{(ab)c = a(bc)}$

Solution.

$Associative\ property\ w.r.t\ Multiplication.$

(iii). $\boldsymbol{7 \times 1 = 7}$

Solution.

$Multiplicative\ identity.$

(iv). $\boldsymbol{x > y\ or\ x = y\ or\ x < y}$

Solution.

$Trichotromy\ Property.$

(v). $\boldsymbol{ab = ba}$

Solution.

$Commutative\ property\ w.r.t\ Multiplication.$

(vi). $\boldsymbol{a + b = b + c => a = b}$

Solution.

$Cancellation\ Law\ w.r.t\ Addition.$

(vii). $\boldsymbol{5 + (-5) = 0}$

Solution.

$Additive\ Inverse.$

(viii). $\boldsymbol{7 \times \frac{1}{7}}$

Solution.

$Multiplicative\ Inverse.$

(ix). $\boldsymbol{a > b =>\ \ ac > bc\ (\ c > 0)}$

Solution.

$Multplicative\ propert.$

Question.2. Fill in the following blanks by stating the properties of real numbers used.

$$\boldsymbol{3x + 3(y - x)}$$

Solution.

Given

$3x + 3(y - x) = 3x + 3y - 3x$

$Distributive\ property\ w.r.t\ muliplication\ over\ subtraction.$

$= 3x - 3x + 3y \quad commutative\ property\ w.r.t\ addition.$

$= 0 + 3y \quad additive\ inverse\ property.$

$= 3y \quad additive\ identity.$

Answer.

Question.3. Give the name of property used in the following.

(i). $\boldsymbol{\sqrt{24} + 0 = \sqrt{24}}$

Solution.

$Additive\ identity.$

(ii). $\boldsymbol{-\frac{2}{3}\left(5 + \frac{7}{2}\right) = \left(-\frac{2}{3}\right)(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)}$

Solution.

$Distributive\ property\ of\ multiplication\ over\ addition.$

(iii). $\boldsymbol{\pi + (-\pi) = 0}$

Solution.

$Additive\ Inverse.$

(iv). $\boldsymbol{\sqrt{3}.\sqrt{3}\ is\ a\ real\ number}.$

Solution.

$Closure\ law\ for\ multiplication.$

(v). $\boldsymbol{\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right) = 1}$

Solution.

$Multiplicative\ Inverse.$

Radicals and Radicands:

If n is a positive integer greater than 1 and a is a real number , then any real number x such that $x^n = a$ is called the nth root of $a\ , and\ in\ symbols\ is\ written\ as$

$$x = \sqrt[n]{a} \quad or\ x = (a)^{\frac{1}{n}}$$

And $\sqrt[n]{a}$ is called radical, the symbol $\sqrt{\ \ }$ is called the radical sign , n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Radical and Exponent Form:

$x = \sqrt[n]{a}\ is\ called\ redical\ form\ and\ a = x^{\frac{1}{n}}\ is\ called\ exponent\ form.$

Some Properties of Radicals:

(i). $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

(ii). $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iii). $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

(iv). $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(v). $\sqrt[n]{a^n} = a$

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Exercise # 2.3

Question.1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i). $\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$

Solution.

$$\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$$

(ii). $2^{\frac{3}{5}}$

Solution.

$$2^{\frac{3}{5}} = \sqrt[5]{2^3}$$

(iii). $-7^{\frac{1}{3}}$

Solution.

$$-7^{\frac{1}{3}} = -\sqrt[3]{7}$$

(iv). $y^{-\frac{2}{3}}$

Solution.

$$y^{-\frac{2}{3}} = \sqrt[3]{y^{-2}}$$

Question.2. Tell whether the following statements are true or false?

(i). $5^{\frac{1}{5}} = \sqrt{5}$

Solution.

$$False\ because\ 5^{\frac{1}{5}} = \sqrt[5]{5}\ is\ true.$$

(ii). $2^{\frac{2}{3}} = \sqrt[3]{4}$

Solution.

$$True\ because\ 2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4}\ is\ true.$$

(iii). $\sqrt{49} = \sqrt{7}$

Solution.

$$False\ because\ \sqrt{49} = \sqrt{7^2} = 7 is\ true.$$

(iv). $\sqrt[3]{x^{27}} = x^3$

Solution.

$$False\ because\ \sqrt[3]{x^{27}} = x^{\frac{27}{3}} = x^9\ is\ true.$$

Question.3. Simplify the following radical expressions.

(i). $\sqrt[3]{-125}$

Solution.

$$\sqrt[3]{-125} = \sqrt[3]{-5^3}$$
$$= (-5)^{3\times\frac{1}{3}}$$
$$= -5$$

Answer.

(ii). $\sqrt[4]{32}$

Solution.

$$\sqrt[4]{32} = \sqrt[4]{2^4 \times 2}$$
$$= (2^4 \times 2)^{\frac{1}{4}}$$
$$= (2^4)^{\frac{1}{4}} \times 2^{\frac{1}{4}}$$
$$= 2 \times \sqrt[4]{2}$$
$$= 2\sqrt[4]{2}$$

Answer.

(iii). $\sqrt[5]{\frac{3}{32}}$

Solution.

$$\sqrt[5]{\frac{3}{32}} = \left(\frac{3}{32}\right)^{\frac{1}{5}}$$
$$= \left(\frac{3}{2^5}\right)^{\frac{1}{5}}$$
$$= \frac{3^{\frac{1}{5}}}{2^{5\times\frac{1}{5}}}$$
$$= \frac{\sqrt[5]{3}}{2}$$

Answer.

(iv). $\sqrt[3]{\frac{-8}{27}}$

Solution.

$$\sqrt[3]{\frac{-8}{27}} = \left(\frac{-2^3}{3^3}\right)^{\frac{1}{3}}$$
$$= \frac{-2^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}}$$
$$= \frac{-2}{3}$$

Answer.

Base and Exponents:

In the exponential form a^n ($read\ as\ a\ to\ the\ nth\ power$) we call $''a''$ as the base and $''n''$ as the exponent or power.

Laws of Exponents:

If $a, b \in R\ and\ m\ , n\ are\ positive\ integers, then$

(i). $a^m \,.\, a^n = a^{m+n}$

(ii). $(a^m)^n = a^{mn}$

(iii). $(ab)^n = a^n b^n$

(iv). $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(v). $\frac{a^m}{a^n} = a^{m-n}$

(vi). $a^0 = 1\ , where\ a \neq 0$

(vii). $a^{-n} = \frac{1}{a^n}\ , where\ a \neq 0$

Exercise # 2.4

Question.1. Use laws of exponents to simplify

(i). $\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

Solution.

$$\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}} = \frac{(3^5)^{-\frac{2}{3}}(2^5)^{-\frac{1}{5}}}{(14^2)^{-1\times\frac{1}{2}}}$$

$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{14^{-1}}$$

$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{(2\times 7)^{-1}}$$

$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{2^{-1}\times 7^{-1}}$$

$$= \frac{3^{-\frac{10}{3}}}{7^{-1}}$$

$$= \frac{7}{3^{\frac{9+1}{3}}}$$

$$= \frac{7}{3^{\frac{9}{3}+\frac{1}{3}}}$$

$$= \frac{7}{3^{\frac{9}{3}}\times 3^{\frac{1}{3}}}$$

$$= \frac{7}{3^3\times\sqrt[3]{3}}$$

$$= \frac{7}{27\sqrt[3]{3}}$$

Answer.

(ii). $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution.

$$(2x^5y^{-4})(-8x^{-3}y^2) = (2)(-8)x^5.y^{-4}.x^{-3}y^2$$

$$= -16\,x^{5-3}.y^{-4+2}$$

$$= -16\,x^2.y^{-2}$$

$$= -\frac{16x^2}{y^2}$$

Answer.

(iii). $\left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

Solution.

$$\left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3} = \left(\frac{y^{-1+3}}{x^{4+2}z^{0+4}}\right)^{-3}$$

$$= \left(\frac{y^2}{x^6z^4}\right)^{-3}$$

$$= \left(\frac{x^6z^4}{y^2}\right)^{3}$$

$$= \frac{x^{6\times3}z^{4\times3}}{y^{2\times3}}$$

$$= \frac{x^{18}z^{12}}{y^6}$$

Answer.

(iv). $\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$

Solution.

$$\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$$

$$= \frac{(3^4)^n.3^5-(3)^{4n-1}(3)^5}{(3^2)^{2n}.3^3}$$

$$= \frac{(3)^{4n}.3^5-(3)^{4n-1}(3)^5}{(3)^{4n}.3^3}$$

$$= \frac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}}$$

$$= \frac{3^{4n+5}-3^{4n+4}}{3^{4n+3}}$$

$$= \frac{3^{4n+4}(3^1-1)}{3^{4n+3}}$$

$$= 3^{4n+4-4n-3}(2)$$

$$= 3^1(2)$$

$$= 6$$

Answer

Question.2. Show that

$$\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

Solution.

$$L.H.S = \left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}$$

$$L.H.S = (x^{a-b})^{a+b}\times(x^{b-c})^{b+c}\times(x^{c-a})^{c+a}$$

$$L.H.S = x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$$

$$L.H.S = x^{a^2-b^2}\times x^{b^2-c^2}\times x^{c^2-a^2}$$

$$L.H.S = x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$L.H.S = x^0 = 1$$

Hence Proved.

Question.3. Simplify

(i). $\frac{2^{\frac{1}{3}}(27)^{\frac{1}{3}}(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}(4)^{-\frac{1}{3}}9^{\frac{1}{4}}}$

Solution.

$$\frac{2^{\frac{1}{3}}(27)^{\frac{1}{3}}(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}(4)^{-\frac{1}{3}}9^{\frac{1}{4}}} = \frac{2^{\frac{1}{3}}(3^3)^{\frac{1}{3}}(2^2.3.5)^{\frac{1}{2}}}{(2^2.3^2.5)^{\frac{1}{2}}(2^2)^{-\frac{1}{3}}(3^2)^{\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}}3^1.2^{2\times\frac{1}{2}}.3^{\frac{1}{2}}.5^{\frac{1}{2}}}{2^{2\times\frac{1}{2}}.3^{2\times\frac{1}{2}}.5^{\frac{1}{2}}.2^{-\frac{2}{3}}3^{2\times\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}}3^1.2^1.3^{\frac{1}{2}}.5^{\frac{1}{2}}}{2^1.3^1.5^{\frac{1}{2}}.2^{-\frac{2}{3}}3^{\frac{1}{2}}}$$

$$= \frac{2^{\frac{1}{3}}}{2^{-\frac{2}{3}}}$$
$$= 2^{\frac{1}{3}+\frac{2}{3}}$$
$$= 2^{\frac{1+2}{3}}$$
$$= 2^{\frac{3}{3}}$$
$$=2$$
Answer.

(ii). $\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$

Solution.

$$\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}} = \sqrt{\frac{(6^3)^{\frac{2}{3}}(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2\,.\,5^1}{\left(\frac{1}{25}\right)^{-\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2\,.\,5^1}{(25)^{\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2\,.5}{(5^2)^{\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2.5}{5}}$$
$$= \sqrt{36}$$
$$= 6$$
Answer.

(iii). $5^{2^3} \div (5^2)^3$

Solution.

$$5^{2^3} \div (5^2)^3 = \frac{5^8}{5^6}$$
$$= 5^{8-6}$$
$$= 5^2$$
$$= 25$$
Answer.

(iv). $(x^3)^2 \div x^{3^2}$

Solution.

$$(x^3)^2 \div x^{3^2} = \frac{x^6}{x^8}$$
$$= \frac{1}{x^{8-6}}$$
$$= \frac{1}{x^2}$$
Answer.

Complex Numbers:
The numbers of the form $x + iy$, where $x, y \in \Re$,are called **complex numbers,** here x is called **real part** and y is called **imaginary part** of the complex number.

Remarks:

1. Every real number is a complex number with 0 as its imaginary part.

Conjugate Complex Numbers:
$if\ Z = a + ib\ be\ a\ complex\ number\ then\ \bar{Z} = a - ib$ is the conjugate of the complex number $Z = a + ib$.

Remarks:

1. A real number is self-Conjugate.

Equality of Two Complex Numbers:
Two complex numbers $a + bi\ and\ c + di\ are\ said\ to\ be\ equal\ if\ a = c\ and\ b = d.$
That is
$$a + ib = c + id \quad \Rightarrow a = b\ and\ c = d.$$

Exercise # 2.5

Question.1. Evaluate

(i). $\boldsymbol{i^7}$

Solution.

$$\boldsymbol{i^7 = i^6.i}$$
$$\boldsymbol{= (i^2)^3.i}$$
$$\boldsymbol{= (-1)^3.i}$$
$$\boldsymbol{= (-1).i}$$
$$\boldsymbol{= -i}$$
Answer.

(ii). $\boldsymbol{i^{50}}$

Solution.

$$\boldsymbol{i^{50} = (i^2)^{25}}$$
$$\boldsymbol{= (-1)^{25}}$$
$$\boldsymbol{= -1}$$
Answer.

$\boldsymbol{(iii).\,i^{12}}$

Solution.

$$\boldsymbol{i^{12} = (i^2)^6}$$
$$\boldsymbol{= (-1)^6}$$
$$\boldsymbol{= 1}$$
Answer.

(iv). $\boldsymbol{(-i)^8}$

Solution.

$$\boldsymbol{(-i)^8 = i^8}$$
$$\boldsymbol{= (i^2)^4}$$
$$\boldsymbol{= (-1)^4}$$
$$\boldsymbol{= 1}$$
Answer.

(v). $\boldsymbol{(-i)^5}$

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Solution.

$$(-i)^5 = -i^5$$
$$= -i^4.i$$
$$= -(i^2)^2.i$$
$$= -(-1)^2.i$$
$$= -(1).i$$
$$= -i$$

Answer.

(vi). i^{27}

Solution.

$$i^{27} = i^{26}.i$$
$$= (i^2)^{13}.i$$
$$= (-1)^{13}.i$$
$$= (-1).i$$
$$= -1$$

Answer.

Question.2. Write the conjugate of the following numbers.

(i). $2 + 3i$

Solution.

$$Suppose\ \ Z = 2 + 3i$$
$$\bar{Z} = \overline{2 + 3i} = 2 - 3i$$

Answer.

(ii). $3 - 5i$

Solution.

$$Suppose\ \ Z = 3 - 5i$$
$$\bar{Z} = \overline{3 - 5i} = 3 + 5i$$

Answer.

(iii). $-i$

Solution.

$$Suppose\ \ Z = -i$$
$$\bar{Z} = \overline{-i} = +i$$

Answer.

(iv). $-3 + 4i$

Solution.

$$Suppose\ \ Z = -3 + 4i$$
$$\bar{Z} = \overline{-3 + 4i} = -3 - 4i$$

Answer.

(v). $-4 - i$

Solution.

$$Suppose\ \ Z = -4 - i$$
$$\bar{Z} = \overline{-4 - i} = -4 + i$$

Answer.

Question.3. Write the real and imaginary part of the following numbers.

(i). $1 + i$

Solution.

$$Suppose\ \ Z = 1 + i$$
$$Re(Z) = 1 \quad , \qquad Im(Z) = 1$$

Answer.

(ii). $-1 + 2i$

Solution.

$$Suppose\ \ Z = -1 + 2i$$
$$Re(Z) = -1 \quad , \qquad Im(Z) = 2$$

Answer.

(iii). $-3i + 2$

Solution.

$$Suppose\ \ Z = -3i + 2 = 2 - 3i$$
$$Re(Z) = 2 \quad , \qquad Im(Z) = -3$$

Answer.

(iv). $-2 - 2i$

Solution.

$$Suppose\ \ Z = -2i - 2 = -2 - 2i$$
$$Re(Z) = -2 \quad , \qquad Im(Z) = -2$$

Answer.

(v). $-3i$

Solution.

$$Suppose\ \ Z = -3i = 0 - 3i$$
$$Re(Z) = 0 \quad , \qquad Im(Z) = -3$$

Answer.

(vi). $2 + 0i$

Solution.

$$Suppose\ \ Z = 2 + 0i$$
$$Re(Z) = 2 \quad , \qquad Im(Z) = 0$$

Answer.

Question.4. Find the value of x and y if

$$x + iy + 1 = 4 - 3i$$

Solution.

Given that

$$x + 1 + iy = 4 - 3i$$

Separating real and imaginary parts

$$x + 1 = 4 \quad , \quad y = -3$$
$$x = 4 - 1 \quad , \quad y = -3$$
$$x = 3 \quad , \quad y = -3$$

Answer.

Operations on Complex Numbers:

The symbols a,b,c,d,k, where used, represent real numbers

Addition of Two Complex Numbers:

$$(a + ib) + (c + id) = (a + b) + i(c + d).$$

Scalar Multiplication:

$$k(a + ib) = ka + ikb.$$

Subtraction of Two Complex Numbers:

$$(a + ib) - (c + id) = (a - b) + i(c - d).$$

Multiplication of Two Complex Numbers:

$$(a + ib)(c + id) = (ac - bd) + i(ad + bc).$$

Division of two Complex Numbers:

$$\frac{(a + ib)}{(c + id)} = \frac{ac - bd}{c^2 + d^2} + i\frac{bc - ad}{c^2 + d^2}$$

Exercise # 2.6

Question.1. Identify the following statements as true or false.

(i). $\sqrt{-3}\times\sqrt{-3}=3$

Solution.

$$False\ because\ \sqrt{-3}\times\sqrt{-3}=\sqrt{3}i\times\sqrt{3}i$$
$$=\left(\sqrt{3}\right)^2 i^2=-3$$

(ii). $i^{73}=-i$

Solution.

$$False\ because\ i^{73}=i^{72}.i=(i^2)^{36}.i$$
$$=(-1)^{36}.i=i$$

(iii). $i^{10}=-1$

Solution.

$$True\ because\ i^{10}=(i^2)^5=(-1)^5=-1$$

(iv). $Complex\ conjugate\ of\ (-6i+i^2)\ is\ (-1+6i)$

Solution.

$$True\ because\ \overline{-6i+i^2}=\overline{-6i-1}=-1+6i$$

(v). Difference of a complex number $z=a+bi\ and\ its\ conjugate\ is\ a\ real\ number.$

Solution.

$$False\ because\ Z-\overline{Z}=(a+bi)-(a-bi)$$
$$=a+bi-a+bi=2bi$$

(vi). If $(a-1)-(b+3)i=5+8i\ then\ a=6\ and\ \ b=-11.$

Solution.

True because Comparing real and imaginary parts in given equation

$$a-1=5\quad,\quad -(b+3)=8$$
$$a=5+1\quad,\quad b+3=-8$$
$$a=6\quad,\quad b=-8-3$$
$$a=6\quad,\quad b=-11$$

(vii) Product of a complex number and its conjugate is always a non-negative real number.

Solution.

$$\boldsymbol{True\ because\ for\ a\ complex\ numer\ Z=a+bi}$$
$$\boldsymbol{Z.\overline{Z}=(a+bi).(a-bi)=a^2-(bi)^2}$$
$$\boldsymbol{=a^2+b^2}$$

Is a real number.

Question.2. Express each complex number in the standard form $a+bi$ where 'a' and 'b' are real numbers.

(i). $(2+3i)+(7-2i)$

Solution.

$$(2+3i)+(7-2i)=2+3i+7-2i$$
$$=\boldsymbol{9+i}$$

Answer.

(ii). $\boldsymbol{2(5+4i)-3(7+4i)}$

Solution.

$$\boldsymbol{2(5+4i)-3(7+4i)=10+8i-21-12i}$$
$$\boldsymbol{=-11-3i}$$

Answer.

(iii). $\boldsymbol{-1(-3+5i)-(4+9i)}$

Solution.

$$\boldsymbol{-1(-3+5i)-(4+9i)=3-5i-4-9i}$$
$$\boldsymbol{=-1-14i}$$

Answer.

(iv). $\boldsymbol{2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}}$

Solution.

$$\boldsymbol{2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}}$$
$$\boldsymbol{=2(-1)+6i^2i+3i^{16}-6i^{18}i+4i^{24}i}$$
$$\boldsymbol{=2(-1)+6(-1)i+3(i^2)^8-6(i^2)^9i+4(i^2)^{12}i}$$
$$\boldsymbol{=-2-6i+3(-1)^8-6(-1)^9i+4(-1)^{12}i}$$
$$\boldsymbol{=-2-6i+3(1)-6(-1)i+4(1)i}$$
$$\boldsymbol{=-2-6i+3+6i+4i}$$
$$\boldsymbol{=1+4i}$$

Question.3. Simplify and write your answer in the form $a+bi$.

(i). $\boldsymbol{(-7+3i)(-3+2i)}$

Solution.

$$\boldsymbol{(-7+3i)(-3+2i)=21-14i-9i+6i^2}$$
$$\boldsymbol{=21-14i-9i-6}$$
$$\boldsymbol{=15-23i}$$

Answer.

(ii). $\boldsymbol{(2-\sqrt{-4})(3-\sqrt{-4})}$

Solution.

$$\boldsymbol{(2-\sqrt{-4})(3-\sqrt{-4})=(2-2i)(3-2i)}$$
$$\boldsymbol{=2(3-2i)-2i(3-2i)}$$
$$\boldsymbol{=6-4i-6i+4i^2}$$
$$\boldsymbol{=6-10i-4}$$
$$\boldsymbol{=2-10i}$$

Answer.

(iii). $\boldsymbol{\left(\sqrt{5}-3i\right)^2}$

Solution.

$$\boldsymbol{\left(\sqrt{5}-3i\right)^2=\left(\sqrt{5}\right)^2+(3i)^2-2(\sqrt{5})(3i)}$$
$$\boldsymbol{=5+9i^2-6\sqrt{5}i}$$
$$\boldsymbol{=5-9-6\sqrt{5}i}$$
$$\boldsymbol{=-4-6\sqrt{5}i}$$

Answer.

(iv). $\boldsymbol{(2-3i)(\overline{3-2i})}$

Solution.

$$\boldsymbol{(2-3i)(\overline{3-2i})=(2-3i)(3+2i)}$$
$$\boldsymbol{=2(3+2i)-3i(3+2i)}$$
$$\boldsymbol{=6+4i-9i-6i^2}$$
$$\boldsymbol{=6-5i+6}$$

$$= 12 - 5i$$

Answer.

Question.4. Simplify and write your answer in the form of $a + bi$.

(i). $-\frac{2}{1+i}$

Solution.

$$-\frac{2}{1+i} = \frac{-2}{1+i} \times \frac{1-i}{1-i}$$
$$= \frac{-2+2i}{1^2 - i^2}$$
$$= \frac{-2+2i}{1+1}$$
$$= \frac{-2+2i}{2}$$
$$= -\frac{2}{2} + \frac{2i}{2}$$
$$= -1 + i$$

Answer.

(ii). $\frac{2+3i}{4-i}$

Solution.

$$\frac{2+3i}{4-i} = \frac{2+3i}{4-i} \times \frac{4+i}{4+i}$$
$$= \frac{2(4+i) + 3i(4+i)}{4^2 - i^2}$$
$$= \frac{8 + 2i + 12i + 3i^2}{16+1}$$
$$= \frac{8 + 14i - 3}{17}$$
$$= \frac{4 + 14i}{17}$$
$$= \frac{4}{17} + \frac{14}{17}i$$

Answer.

(iii). $\frac{9-7i}{3+i}$

Solution.

$$\frac{9-7i}{3+i} = \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$
$$= \frac{9(3-i) - 7i(3-i)}{3^2 - i^2}$$
$$= \frac{27 - 9i - 21i + 7i^2}{9+1}$$
$$= \frac{27 - 30i - 7}{10}$$
$$= \frac{20 - 30i}{10}$$
$$= \frac{20}{10} - \frac{30}{10}i$$
$$= 2 - 3i$$

Answer.

(iv). $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

Solution.

$$\frac{2-6i}{3+i} - \frac{4+i}{3+i} = \frac{2-6i}{3+i} \times \frac{3-i}{3-i} - \frac{4+i}{3+i} \times \frac{3-i}{3-i}$$
$$= \frac{2(3-i) - 6i(3-i)}{3^2 - i^2} - \frac{4(3-i) + i(3-i)}{3^2 - i^2}$$
$$= \frac{6 - 2i - 18i + 6i^2}{9+1}$$
$$- \frac{12 - 4i + 3i - i^2}{9+1}$$
$$= \frac{6 - 20i - 6}{10} - \frac{12 - i + 1}{10}$$
$$= \frac{-20i}{10} - \frac{13 - i}{10}$$
$$= \frac{-20i - 13 + i}{10}$$
$$= \frac{-20i - 13 + i}{10}$$
$$= \frac{-13 - 19i}{10}$$
$$= -\frac{13}{10} - \frac{19}{10}i$$

Answer.

(v). $\left(\frac{1+i}{1-i}\right)^2$

Solution.

$$\left(\frac{1+i}{1-i}\right)^2 = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^2$$
$$= \left(\frac{1(1+i) + i(1+i)}{1^2 - i^2}\right)^2$$
$$= \left(\frac{1 + i + i + i^2}{1+1}\right)^2$$
$$= \left(\frac{1 + 2i - 1}{2}\right)^2$$
$$= \left(\frac{2i}{2}\right)^2$$
$$= i^2$$
$$= -1$$

Answer.

(vi). $\frac{1}{(2+3i)(1-i)}$

Solution.

$$\frac{1}{(2+3i)(1-i)} = \frac{1}{2(1-i) + 3i(1-i)}$$
$$= \frac{1}{2 - 2i + 3i - 3i^2}$$
$$= \frac{1}{2 + i + 3}$$
$$= \frac{1}{5+i}$$
$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$
$$= \frac{5-i}{5^2 - (i)^2}$$

$$=\frac{5-i}{25+1}$$
$$=\frac{5-i}{26}$$
$$=\frac{5}{26}-\frac{i}{26}$$

Answer.

Question.5.

Calculate (a) $\overline{Z}$ (b) $Z+\overline{Z}$ (c) $Z-\overline{Z}$ (d) $Z\overline{Z}$ for each of the following

(i). $Z=-i$

Solution.

(a). $\overline{Z}=\overline{-i}=i$

(b). $Z+\overline{Z}=-i+i=0$

(c). $Z-\overline{Z}=(-i)-(i)=-i-i=-2i$

(d). $Z\overline{Z}=(-i)(i)=-i^2=1$

(ii). $Z=2+i$

Solution.

(a). $\overline{Z}=\overline{2+i}=2-i$

(b). $Z+\overline{Z}=2+i+2-i=4$

(c). $Z-\overline{Z}=(2+i)-(2-i)=2+i-2+i=2i$

(d). $Z\overline{Z}=(2+i)(2-i)=2^2-i^2=4+1=5$

(iii). $Z=\frac{1+i}{1-i}$

Solution.

$$Z=\frac{1+i}{1-i}$$
$$Z=\frac{1+i}{1-i}\times\frac{1+i}{1+i}$$
$$Z=\frac{1+i+i+i^2}{1+1}$$
$$Z=\frac{1+2i-1}{2}$$
$$Z=\frac{2i}{2}$$
$$Z=i$$

(a). $\overline{Z}=\overline{i}=-i$

(b). $Z+\overline{Z}=i-i=0$

(c). $Z-\overline{Z}=(i)-(-i)=i+i=2i$

(d). $Z\overline{Z}=(i)(-i)=-i^2=1$

(iv). $Z=\frac{4-3i}{2+4i}$

Solution.

$$Z=\frac{4-3i}{2+4i}$$
$$Z=\frac{4-3i}{2+4i}\times\frac{2-4i}{2-4i}$$
$$Z=\frac{8-16i-6i+12i^2}{2^2-(4i)^2}$$
$$Z=\frac{8-22i-12}{4-16i^2}$$
$$Z=\frac{-4-22i}{4+16}$$
$$Z=\frac{-4-22i}{20}$$
$$Z=-\frac{4}{20}-\frac{22}{20}i$$
$$Z=-\frac{1}{5}-\frac{11}{10}i$$

(a). $\overline{Z}=\overline{-\frac{1}{5}-\frac{11}{10}i}=-\frac{1}{5}+\frac{11}{10}i$

(b). $Z+\overline{Z}=-\frac{1}{5}-\frac{11}{10}i+-\frac{1}{5}+\frac{11}{10}i$

$$Z+\overline{Z}=-\frac{1}{5}-\frac{1}{5}=\frac{-1-1}{5}=-\frac{2}{5}=-\frac{2}{5}$$
$$Z+\overline{Z}=-\frac{2}{5}$$

(c). $Z-\overline{Z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)-(-\frac{1}{5}+\frac{11}{10}i)$

$$Z-\overline{Z}=-\frac{1}{5}-\frac{11}{10}i+\frac{1}{5}-\frac{11}{10}i$$
$$Z-\overline{Z}=-\frac{11}{10}i-\frac{11}{10}i$$
$$Z-\overline{Z}=\frac{-11-11}{10}i$$
$$Z-\overline{Z}=-\frac{22}{10}i$$
$$Z-\overline{Z}=-\frac{11}{5}i$$

(d). $Z\overline{Z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)\left(-\frac{1}{5}+\frac{11}{10}i\right)$

$$Z\overline{Z}=\left(-\frac{1}{5}\right)^2-\left(\frac{11}{10}i\right)^2$$
$$Z\overline{Z}=\frac{1}{25}-\frac{121}{100}i^2$$
$$Z\overline{Z}=\frac{1}{25}+\frac{121}{100}$$
$$Z\overline{Z}=\frac{4+121}{100}$$
$$Z\overline{Z}=\frac{125}{100}$$
$$Z\overline{Z}=\frac{5}{4}$$

Answer.

Question.6. If $z=2+3i$, $w=5-4i$, *show that*

(i). $\overline{z+w}=\overline{z}+\overline{w}$

Solution.

$$L.H.S=\overline{z+w}$$
$$L.H.S=\overline{2+3i+5-4i}$$
$$L.H.S=\overline{8-i}$$
$$L.H.S=8+i---(1)$$
$$R.H.S=\overline{z}+\overline{w}$$
$$R.H.S=\overline{2+3i}+\overline{5-4i}$$
$$R.H.S=2-3i+5+4i$$

$$R.H.S = 8 + i - - - (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(ii). $\overline{z - w} = \bar{z} - \bar{w}$

Solution.

$$L.H.S = \overline{z - w}$$
$$L.H.S = \overline{(2 + 3\iota) - (5 - 4\iota)}$$
$$L.H.S = \overline{2 + 3\iota - 5 + 4\iota}$$
$$L.H.S = \overline{-3 + 7\iota}$$
$$L.H.S = -3 - 7i - - - (1)$$
$$R.H.S = \bar{z} - \bar{w}$$
$$R.H.S = (\overline{2 + 3\iota}) - (\overline{5 - 4\iota})$$
$$R.H.S = (2 - 3i) - (5 + 4i)$$
$$R.H.S = 2 - 3i - 5 - 4i$$
$$R.H.S = -3 - 7i - - - (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(iii). $\overline{zw} = \bar{z}\,\bar{w}$

Solution.

$$L.H.S = \overline{zw}$$
$$L.H.S = \overline{(2 + 3\iota)(5 - 4\iota)}$$
$$L.H.S = \overline{10 - 8\iota + 15\iota - 12\iota^2}$$
$$L.H.S = \overline{10 + 7\iota + 12}$$
$$L.H.S = \overline{22 + 7\iota}$$
$$L.H.S = 22 - 7i - - - (1)$$
$$R.H.S = \bar{z}\bar{w}$$
$$R.H.S = (\overline{2 + 3\iota})(\overline{5 - 4\iota})$$
$$R.H.S = (2 - 3i)(5 + 4i)$$
$$R.H.S = 10 + 8i - 15i - 12i^2$$
$$R.H.S = 10 - 7i + 12$$
$$R.H.S = 22 - 7i - - - (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(iv). $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

Solution.

$$L.H.S = \overline{\left(\frac{z}{w}\right)}$$
$$L.H.S = \overline{\left(\frac{2 + 3\iota}{5 - 4\iota}\right)}$$
$$L.H.S = \overline{\left(\frac{2 + 3\iota}{5 - 4\iota} \times \frac{5 + 4\iota}{5 + 4\iota}\right)}$$
$$L.H.S = \overline{\left(\frac{10 + 8\iota + 15\iota + 12\iota^2}{5^2 - (4\iota)^2}\right)}$$
$$L.H.S = \overline{\left(\frac{10 + 23\iota - 12}{25 - 16\iota^2}\right)}$$
$$L.H.S = \overline{\left(\frac{-2 + 23\iota}{25 + 16}\right)}$$
$$L.H.S = \overline{\left(\frac{-2 + 23\iota}{41}\right)}$$
$$L.H.S = \overline{\left(-\frac{2}{41} + \frac{23}{41}\iota\right)}$$
$$L.H.S = -\frac{2}{41} - \frac{23}{41}i - - - (1)$$
$$R.H.S = \frac{\bar{z}}{\bar{w}}$$
$$R.H.S = \frac{(\overline{2 + 3\iota})}{\overline{5 - 4\iota}}$$
$$R.H.S = \frac{2 - 3i}{5 + 4i}$$
$$R.H.S = \frac{2 - 3i}{5 + 4i} \times \frac{5 - 4i}{5 - 4i}$$
$$R.H.S = \frac{10 - 8i - 15i + 12i^2}{5^2 - (4i)^2}$$
$$R.H.S = \frac{10 - 23i - 12}{25 - 16i^2}$$
$$R.H.S = \frac{-2 - 23i}{25 + 16}$$
$$R.H.S = \frac{-2 - 23i}{41}$$
$$R.H.S = -\frac{2}{41} - \frac{23}{41}i - - - (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(v). $\frac{1}{2}(z + \bar{z})$ *is a real part of z.*

Solution.

$$\frac{1}{2}(z + \bar{z}) = \frac{1}{2}\,(2 + 3i + \overline{2 + 3\iota})$$
$$= \frac{1}{2}\,(2 + 3i + 2 - 3i)$$
$$= \frac{1}{2}\,(4)$$
$$= 2 \text{ which is real part of } z.$$

Hence Proved.

(vi). $\frac{1}{2i}(z - \bar{z})$ *is a imaginary part of z.*

Solution.

$$\frac{1}{2i}(z - \bar{z}) = \frac{1}{2i}\,\left((2 + 3i) + (\overline{2 + 3\iota})\right)$$
$$= \frac{1}{2i}\,\left((2 + 3i) - (2 - 3i)\right)$$
$$= \frac{1}{2i}\,(2 + 3i - 2 + 3i)$$
$$= \frac{1}{2i}\,(6i)$$
$$= 3 \text{ which is imaginary part of } z.$$

Hence Proved.

Question.7. Solve the following equations for real x *and* y.

(i). $(2 - 3i)(x + iy) = 4 + i$

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Solution. Given that

$$(2-3i)(x+iy)=4+i$$
$$2(x+iy)-3i(x+iy)=4+i$$
$$2x+2iy-3ix-3i^2y=4+i$$
$$2x+2iy-3ix+3y=4+i$$
$$2x+3y+(2y-3x)i=4+i$$

Comparing real and imaginary parts, we have

$$2x+3y=4---(i)\ ,\quad 2y-3x=1---(ii)$$

$3\times(i)+2\times(ii)$, *we have*

$$3(2x+3y)+2(2y-3x)=3(4)+2(1)$$
$$6x+9y+4y-6x=12+2$$
$$13y=14$$
$$y=\frac{14}{13}$$

Using value of y *in equation* (i), *we have*

$$2x+3\left(\frac{14}{13}\right)=4$$
$$2x+\frac{42}{13}=4$$
$$2x=4-\frac{42}{13}$$
$$2x=\frac{4\times 13-42}{13}$$
$$2x=\frac{52-42}{13}$$
$$2x=\frac{10}{13}$$
$$x=\frac{10}{13\times 2}$$
$$x=\frac{5}{13}$$

Hence required $x=\frac{5}{13}$ *and* $y=\frac{14}{13}$.

(ii). $(3-2i)(x+iy)=2(x-2yi)+2i-1$

Solution. Given that

$$(3-2i)(x+iy)=2(x-2yi)+2i-1$$
$$3(x+iy)-2i(x+iy)=2x-4yi+2i-1$$
$$3x+3iy-2ix-2i^2y=2x-1+2i-4yi$$
$$3x+3iy-2ix+2y=2x-1+(2-4y)i$$
$$3x+2y+(3y-2x)i=2x-1+(2-4y)i$$

Comparing real and imaginary parts, we have

$$3x+2y=2x-1\ ,(3y-2x)=2-4y$$
$$3x-2x+2y=-1\ ,-2x+3y+4y=2$$
$$x+2y=-1---(i)\ ,-2x+7y=2---(ii)$$

$2\times(i)+(ii)$, *we have*

$$2(x+2y)+(-2x+7y)=2(-1)+2$$
$$2x+4y-2x+7y=-2+2$$
$$11y=0$$
$$y=0$$

Using value of y *in equation* (i), *we have*

$$x+2(0)=-1$$
$$x=-1$$

Hence required $x=-1$ *and* $y=0$.

(iii). $(3+4i)^2-2(x-iy)=x+yi$

Solution. Given that

$$(3+4i)^2-2(x-iy)=x+yi$$
$$(3)^2+(4i)^2+2(3)(4i)-2x+2iy=x+yi$$
$$9+16i^2+12i-2x+2iy=x+yi$$
$$9-16+12i-2x+2iy=x+yi$$
$$-7-2x+(12+2y)i=x+yi$$

Comparing real and imaginary parts, we have

$$-7-2x=x\ ,12+2y=y$$
$$x+2x=-7=\ ,2y-y=12$$
$$3x=-7\ ,y=12$$
$$x=-\frac{7}{3}\ ,y=12$$

Hence required $x=-\frac{7}{3}$ *and* $y=12$.

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MATHEMATICS 9th SCIENCE GROUP

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Chapter 3.

LOGARITHM

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Contents

Scientific Notation:
A number written in the form $a \times 10^n$, $where\ 1 \le a \le 10$ and n is an integer, is called the scientific notation.
Example:
Write each of the following ordinary numbers in scientific notation.
Solution:
(i) $30600 = 3.06 \times 10^4$
$(move\ decimal\ point\ four\ places\ to\ the\ left)$
(ii) $0.000058 = 5.8 \times 10^{-5}$
$(move\ dicemal\ point\ five\ places\ to\ the\ right)$
Example:
Change each of the following numbers from scientific notation to ordinary notation.
Solution:
(i) $6.35 \times 10^6 = 6350000$
$(Move\ the\ decimal\ point\ six\ places\ to\ the\ right)$
(ii) $7.61 \times 10^{-4} = 0.000761$
$(Move\ the\ decimal\ point\ four\ places\ to\ the\ left)$

Exercise 3.1

Question.1. Express each of the following numbers in scientific notation.

(i). 5700
Solution.
$5700 = 5 \wedge 700.$
$$5700 = 5.7 \times 10^3$$
Answer.

(ii). $49,800,000$
Solution.
$49,800,000 = 4 \wedge 9,800,000.$
$$49,800,000 = 4.98 \times 10^7$$
Answer.

(iii). $96,000,000$
Solution.
$96,000,000 = 9 \wedge 6,000,000.$
$$96,000,000 = 9.6 \times 10^7$$
Answer.

(iv). 416.9
Solution.
$416.9 = 4 \wedge 16.9$
$$416.9 = 4.169 \times 10^2$$
Answer.

(v). $83,000$
Solution.
$83,000 = 8 \wedge 3,000.$
$$83,000 = 8.3 \times 10^4$$
Answer.

(vi). 0.00643
Solution.
$0.00643 = 0.006 \wedge 43$
$$0.00643 = 6.43 \times 10^{-3}$$
Answer.

(vii). 0.0074
Solution.
$0.0074 = 0.007 \wedge 4$
$$0.0074 = 7.4 \times 10^{-3}$$
Answer.

(viii). $60,000,000$
Solution.
$60,000,000 = 6 \wedge 0,000,000.$
$$60,000,000 = 6.0 \times 10^7$$
Answer.

(ix). 0.00000000395
Solution.
$0.00000000395 = 0.000000003 \wedge 95$
$$0.00000000395 = 3.95 \times 10^{-9}$$
Answer.

(x). $\frac{275,000}{0.0025}$
Solution.
$\frac{275,000}{0.0025} = \frac{2\wedge75000.}{0.002\wedge5}$
$$\frac{275,000}{0.0025} = \frac{2.75 \times 10^5}{2.5 \times 10^{-3}}$$
$$= \frac{2.75}{2.5} \times 10^5 \times 10^3$$
$$= 1.1 \times 10^{5+3}$$
$$= 1.1 \times 10^8$$
Answer.

Question.2. Express the following numbers in ordinary notation.

(i). 6×10^{-4}
Solution.
$$6 \times 10^{-4} = \frac{6}{10^4}$$
$$= \frac{6}{10000}$$
$$= 0.0006$$
Answer.

(ii). 5.06×10^{10}
Solution.
$$5.06 \times 10^{10} = 5.06 \times 10,000,000,000$$
$$= 50,600,000,000.$$
Answer.

(iii). 9.018×10^{-6}
Solution.
$$9.018 \times 10^{-6} = \frac{9.018}{10^6}$$
$$= \frac{9.018}{1,000,000}$$
$$= 0.000009018$$
Answer.

(iv). 7.865×10^8
Solution.

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$$7.865 \times 10^8 = 7.865 \times 100,000,000$$
$$= 786,500,000.$$

Answer.

Logarithm of real numbers:

if $a^x = y$ *then* x *is called the logarithm of* y *to the base "a" and its written as* $\log_a y = x$ *where* $a > 0, a \neq 1$ *and* $y > 0$

i.e the logarithm of a number y to the base "a" is the index x of the power to which a must be raised to get that number y.

the relation $a^x = y$ *and* $\log_a y = x$ *are equivalen*

When one relation is given, it can be converted into the other. Thus

$$a^x = y \iff \log_a y = x$$

Example: find $log_4 2$ i.e. find log of 2 to the Base 4.

Solution:

Let $\log_4 2 = x$

$$then\ its\ exponenenial\ form\ is\ 4^x = 2$$
$$i.e\ \ 2^{2x} = 2^1 \Rightarrow 2x = 1$$
$$\because x = \frac{1}{2} \Rightarrow log_4 2 = \frac{1}{2}$$

Deductions from Definition of logarithm

1. Since $a^0 = 1,\ log_a 1 = 0$
2. $Since\ a^1 = a\ , log_a\ a = 1$

Common logarithm:

If the base of logarithm is taken as 10 then logarithm is called Common Logarithm.

Characteristic:

The integral part of the logarithm of any number is called the characteristic.

Mantissa: the fractional part of the logarithm of a number is called the mantissa. Mantissa is always positive.

Example: find the mantissa of the logarithm of 43.254

Solution:

Rounding off 43.254 *we consider only the four* Significant digits4325.

(i) We first locate the row corresponding to 43 in the log tables and
(ii) Proceed horizontally till we reach the column corresponding to 2. The number at the intersection is 6355.
(iii) Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row. We get the number 5 at the intersection.
(iv) Adding the two numbers 6355 and 5 we get .6360 as the mantissa of the logarithm of 43.25

Example:

Find the mantissa of the logarithm of 0.002347

Solution:

Here also, we consider only the four Signiant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and proceeding to 4 the resulting number 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.0023476 as o.3705

Example:

1. **Find log 278.23**
2. **Log0.07058**

Solution:

1. $278.22\ can\ be\ rounded\ off\ as\ 278.22$

The characteristic is 2 and the mantissa, using log tables, is .4443

$$\because log278.23 = 2.4443$$

2. The characteristic of $logo.07058\ is - 2$ which is written as $\bar{2}$ by convention.

Using log tables the mantissa is .8487, so that

$$log0.07058 = \bar{2}.8487$$

Example:

Find the numbers whose logarithms are

(i) 1.3247
(ii) $\bar{2}.1324$

Solution:

(i) 1.3247

antilog$1.3247 = 21.12$

(ii) $\bar{2}.1324$

$$antilog(\bar{2}.1324)\ \ is\ \ 0.01356$$

Example 3.2

Question.1. Find the common logarithm of the following numbers:

(i). 232.92

Solution.

$$\boldsymbol{Characteristics = 2}$$
$$\boldsymbol{Mantisa = 0.3672}$$
$$\boldsymbol{Log(232.92) = 2.3672}$$

Answer.

(ii). 29.326

Solution.

$$\boldsymbol{Characteristics = 1}$$
$$\boldsymbol{Mantisa = 0.4672}$$
$$\boldsymbol{Log(29.326) = 1.4672}$$

Answer.

(iii). 0.00032

Solution.

$$\boldsymbol{Characteristics = -4}$$
$$\boldsymbol{Mantisa = 0.5051}$$
$$\boldsymbol{Log(0.00032) = \bar{4}.5051}$$

Answer.

(iv). 0.3206

Solution.

$$\boldsymbol{Characteristics = -1}$$
$$\boldsymbol{Mantisa = 0.5059}$$
$$\boldsymbol{Log(0.3206) = \bar{1}.5059}$$

Answer.

Question.2. If $log 31.09 = 1.4926$, find the values of the following

(i). $log 3.109$

Solution.

$$Characteristics = 0$$
$$Mantisa = 0.4926$$
$$Log(3.109) = 0.4926$$

Answer.

(ii). $log 310.9$

Solution.

$$Characteristics = 2$$
$$Mantisa = 0.4926$$
$$Log(310.9) = 2.4926$$

Answer.

(iii). $log 0.003109$

Solution.

$$Characteristics = -3$$
$$Mantisa = 0.4926$$
$$Log(0.003109) = \bar{3}.4926$$

Answer.

(iv). $log 0.3109$

Solution.

$$Characteristics = -1$$
$$Mantisa = 0.4926$$
$$Log(0.3109) = \bar{1}.4926$$

Answer.

Question.3. Find the number whose common logarithms are:

(i). 3.5621

Solution.

Since it is log of any number. So,

$$Characteristics = 3$$
$$Mantisa = 0.5621$$
$$Mantisa\ in\ antilog = 3.6484$$

Characteristics change the place of decimal.

So

$$Anti - log(3.5621) = 3648.4$$

Answer.

(ii). $\bar{1}.7427$

Solution.

Since it is log of any number. So,

$$Characteristics = -1$$
$$Mantisa = 0.7427$$
$$Mantisa\ in\ antilog = 5.5297$$

Characteristics change the place of decimal.

So

$$Anti - log(\bar{1}.7427) = 0.5530$$

Answer.

Question.4. what replacement for the unknown in each of the following will make the statement true?

(i). $log_3 81 = L$

Solution.

$$log_3 81 = L$$

Exponential Form

$$3^L = 81$$
$$3^L = 3^4$$
$$=> L = 4.$$

(ii). $log_a 6 = 0.5$

Solution.

$$log_a 6 = 0.5$$

Exponential Form

$$a^{0.5} = 6$$
$$a^{\frac{1}{2}} = 6$$

Squaring on both sides , we have

$$\left(a^{\frac{1}{2}}\right)^2 = 6^2$$
$$a = 36.$$

(iii). $log_5 n = 2$

Solution.

$$log_5 n = 2$$

Exponential Form

$$5^2 = n$$
$$25 = n$$
$$n = 25.$$

(iv). $10^p = 40$

Solution.

$10^p = 40$

Logarithm Form

$$log_{10} 40 = p$$
$$p = log_{10} 40$$
$$p = 1.6021$$

Question.5. Evaluate

(i). $log_2 \frac{1}{128}$

Solution.

Let

$$x = log_2 \frac{1}{128}$$

Exponential Form

$$2^x = \frac{1}{128}$$
$$2^x = \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$
$$2^x = 2^{-7}$$
$$=\gg x = -7$$

Answer.

(ii). $log\ 512\ to\ the\ base\ 2\sqrt{2}.$

Solution.

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Let

$$x = log_{2\sqrt{2}}512$$

Exponential Form

$$(2\sqrt{2})^x = 512$$

$$\left(2 \times 2^{\frac{1}{2}}\right)^x = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$\left(2^{1+\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{2+1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{3}{2}}\right)^x = 2^9$$

$$2^{\frac{3x}{2}} = 2^9$$

$$==> \frac{3x}{2} = 9$$

$$3x = 18$$

$$x = \frac{18}{3}$$

$$x = 6$$

Answer.

Question.6. Evaluate the value of $''x''$ from the following statements.

(i). $log_2 x = 5$

Solution.

$$log_2 x = 5$$

Exponential Form

$$2^5 = x$$

$$x = 2^5$$

$$x = 2 \times 2 \times 2 \times 2 \times 2$$

$x = 32$**Answer.**

(ii). $log_{81}9 = x$

Solution.

$$log_{81}9 = x$$

Exponential Form

$$81^x = 9$$

$$(9 \times 9)^x = 9$$

$$9^{2x} = 9^1$$

$$==> \quad 2x = 1$$

$$x = \frac{1}{2}$$

Answer.

(iii). $log_{64}8 = \frac{x}{2}$

Solution.

$$log_{64}8 = \frac{x}{2}$$

Exponential Form

$$(64)^{\frac{x}{2}} = 8$$

$$(8 \times 8)^{\frac{x}{2}} = 8$$

$$(8^2)^{\frac{x}{2}} = 8$$

$$8^x = 8^1$$

$$==> \quad x = 1$$

Answer.

(iv). $log_x 64 = 2$

Solution.

$$log_x 64 = 2$$

Exponential Form

$$(x)^2 = 64$$

Taking square root on both sides

$$\sqrt{x^2} = \sqrt{64}$$

$$x = 8$$

Answer.

(v). $log_3 x = 4$

Solution.

$$log_3 x = 4$$

Exponential Form

$$3^4 = x$$

$$x = 3 \times 3 \times 3 \times 3$$

$$x = 81$$

Answer.

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Laws of Logarithm

(i) $\log_a(mn) = \log_a m + \log_a n$
(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
(iii) $\log_a m^n = n\log_a m$
(iv) $\log_a n = \log_b n \times \log_a b$

$$or \ = \frac{\log_b n}{\log_b a}$$

(i)

$$\boldsymbol{\log_a(mn) = \log_a m + \log_a n}$$

Proof:

$$Let \log_a m = x \ \ and \log_a n = y$$

Writing in exponential form

$$a^x = m \ \ and\ a^y = n$$
$$\because \ a^x \times a^y = mn$$
$$i.e\ \ a^{x+y} = mn$$
$$or\ \log_a(mn) = x + y = \log_a m + \log_a n$$
$$hence \log_a(mn) = \log_a m + \log_a n$$

the rule given above is useful in findingthe
Product of two or more numbers using logarithms

Example:
Evaluate 291.3×42.36
Solution:
$let\ x = 291.3 \times 42.36$
Then$logx = log(291.3 \times 42.36)$

$$= log291.3 + log42.36$$
$$(log_a mn = log_a m + log_a n)$$
$$= 2.4643 + 1.6269 = 4.0912$$
$$x = antilog4.0912 = 12340$$

Example:

$$Evaluate\ 0.2913 \times 0.004236$$

Solution:
Let $\ y = 0.2913 \times 0.004236$
then$logy = \log 0.2913 + log0.004236$

$$logy = \bar{1}.4643 + \bar{3}.6269$$
$$logy = \bar{3}.0912$$
$$y = antilog\bar{3}.0912$$
$$y = 0.001234$$

(ii) $\ \log_a\left(\frac{m}{n}\right) = \log_a m.\log_a n$

Solution:
Let $log_a m = x\ \ and\ log_a n = y$
So that $a^x = m\ \ and\ a^y = n$

$$\because\ \frac{a^x}{a^y} = \frac{m}{n} \Rightarrow a^{x-y} = \frac{m}{n}$$
$$i.e\ \ \ \ log_a\left(\frac{m}{n}\right) = x - y = log_a m - log_a n$$

Hence $log_a\left(\frac{m}{n}\right) = log_a m - log_a n$

Note:

(i) $\log_a\left(\frac{m}{n}\right) \neq \frac{\log_a m}{\log_a n}$

(ii) $\log_a m - \log_a n \neq \log_a(m - n)$

(iii) $\log_a\left(\frac{1}{n}\right) = \log_a l - \log_a n = -\log_a n \ldots$

$$\because \log_a 1 = 0$$

Note:

(i) $log_a(mn) \neq log_a m \times log_a n$
(ii) $log_a m + log_a n \neq log_a(m + n)$
(iii) $log_a(mnp) = log_a m + log_a n + log_n p + \ldots$

Example:

$$Evaluate\ \frac{291.3}{42.36}$$
$$let\ \ x = \frac{291.3}{42.36} so\ that\ logx = \log\frac{291.3}{42.36}$$

Then $\log x = log291.33 - log42.36, \ldots$

$$(\log_a\frac{m}{n} = \log_a m - \log_a n)$$
$$\log x = 2.4643 - 1.6269 = 0{,}8374$$

Thus $x = antilogo.8374 = 6.877$

Example:

$$Evaluate\frac{0.0002913}{0.04236}$$

Solution:

$$\boldsymbol{let\ y = \frac{0.0002913}{0.04236}\ \ \ so\ that}$$
$$\boldsymbol{logy = log\left(\frac{0.002913}{0.04236}\right)}$$

Then $logy = log\ o.oo2913 - log0.04236$

$$\boldsymbol{logy = \bar{3}.4643 - \bar{2}.6269}$$
$$\boldsymbol{= \bar{3} + (0.4643 - 0.6269) - \bar{2}}$$
$$\boldsymbol{= \bar{3} - 0.1626 - \bar{2}}$$
$$\boldsymbol{= \bar{3} + (1 - 0.1626) - 1 - \bar{2}}$$

(adding and subtraction 1)

$$\boldsymbol{= \bar{2}.8374}$$
$$\boldsymbol{[\because\ \ \bar{3} - 1 - \bar{2} = -3 - 1 - (-2) = -2 = \bar{2}]}$$

Therefore , $\ y = antilog\bar{2}.8374$

$$\boldsymbol{y = 0.06877}$$

(iii) $lo_a(m^n) = nlog_a m$
Proof:

$$\boldsymbol{let\ log_a m^n = x,\ \ \ i.e\ \ a^x = m^n}$$

And$log_a m = y,\ i.e\ \ \ a^y = m$

$$\boldsymbol{then\ a^x = m^n = (a^y)^n}$$
$$\boldsymbol{i.e\ \ a^x = (a^y)^n = a^{yn} \Rightarrow x = ny}$$
$$\boldsymbol{i.e\ \ \ log_a m^n = nlog_a m}$$

Example:

$$\boldsymbol{Evaluate\ 4\sqrt{(0.0163)^3} = (0.0163)^{\frac{3}{4}}}$$

Solution:

$$\boldsymbol{let\ y = 4\sqrt{(0.0163)^3} = (0.0163)^{\frac{3}{4}}}$$
$$\boldsymbol{loty = \frac{3}{4}(log0.0163)}$$
$$\boldsymbol{= \frac{3}{4} \times \bar{2}.2122}$$
$$\boldsymbol{= \frac{\bar{6}.6366}{4}}$$
$$\boldsymbol{= \frac{\bar{8} + 2.6366}{4}}$$
$$\boldsymbol{= \bar{2} + 0.6592 = \bar{2}.6592}$$

Hence$y = antilog\bar{2}.6592$

$$\boldsymbol{= 0.04562}$$

(iv)Change of base formula:

$$\boldsymbol{log_a n = log_b n \times log_a b\ \ or \frac{log_b n}{log_b a}}$$

Proof:

$let\ log_b n = x\ \ so\ that\ n = b^x$

Taking log to the base $a, we\ have$

$$log_a n = log_a b^x = xlog_a b = log_b n log_a b$$

Thus $log_a n = log_b n log_a b \rightarrow (i)$

Putting $n = a\ in\ the\ above\ result, we\ get$

$$log_b a \times log_a b = log_a^a = 1$$

or $log_a b = \frac{1}{log_b a}$

hence equation$(i)\ gives$

$log_a n = \frac{log_b n}{log_b a} \rightarrow (ii)$

Using the above rule, a natural logarithm can be converted to a common logarithm and vice versa.

$$log_e n = log_{10} n \times log_e 10\ \ or \frac{log_{10} n}{log_{10} e}$$

$$log_{10} n = log_e n \times log_{10} e\ \ or \frac{log_e n}{log_e 10}$$

The valves of $log_e 10\ \ and\ log_{10} e\ \ are\ available$

From the tables.

$log_e 10 = \frac{1}{0.4343} = 2.3026$ and

$log_{10} e = log 2.718 = 0.4343$

Example:

Calculate $\boldsymbol{log_2 3 \times log_3 8}$

Solution:

We know that

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\because\ \log_2 3 \times \log_3 8 = \frac{log3}{log2} \times \frac{log8}{log3}$$

$$\frac{log8}{log2} = \frac{log2^3}{log2}$$

$$= \frac{3log2}{log2} = 333$$

Example3.3

. Which of the following into sum of difference.

(i) $\log(A \times \boldsymbol{B})$

Sol: $\log(A \times \boldsymbol{B}) = \log A + \log B$

(ii) $\log\left(\frac{15.2}{30.5}\right)$

Sol: $\log\left(\frac{15.2}{30.5}\right) = \log 15.2 - \log 30.5$

(iii) $\log\left(\frac{21\times5}{8}\right)$

Sol: $\log\left(\frac{21\times5}{8}\right) = \log 21 + \log 5 - \log 8$

(iv) $\log\sqrt[3]{\frac{7}{15}}$

Sol: $\log\left(\frac{7}{15}\right)^{\frac{1}{3}} = \frac{1}{3}\left(\log\frac{7}{15}\right)$

$$= \frac{1}{3}(\log 7 - \log 15)$$

(v) $\log\frac{(22)^{\frac{1}{3}}}{5^3}$

Sol: $\log\frac{(22)^{\frac{1}{3}}}{5^3} = \log(22)^{\frac{1}{3}} - \log 5^3$

$$\log\frac{(22)^{\frac{1}{3}}}{5^3} = \frac{1}{3}\log 22 - 3\log 5$$

(iii) $\log\left(\frac{25\times47}{29}\right)$

Sol: $\log\left(\frac{25\times47}{29}\right) = \log 25 + \log 47 - \log 29$

Q#2) Express $\log x - 2\ \log x + 3\log(x+1) - \log(x^2-1)$ as a single logarithm.

Sol: $\log x - 2\ \log x + 3\log(x+1) - \log(x^2-1)$

$$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$$

$$= \log\left(\frac{x\ (x+1)^3}{x^2\ (x^2-1)}\right)$$

$$= \log\left(\frac{(x+1)^3}{x\ (x-1)(x+1)}\right)$$

$$= \log\left(\frac{(x+1)^2}{x\ (x-1)}\right)$$

Q#3) Write the following in the single logarithm.

(i) $\log 21 + \log 5$

Sol: $\log 21 + \log 5 = \log(21 \times \mathbf{5})$

(ii) $\log 25 - 2\log 3$

Sol: $\log 2\,5 - 2\log 3 = \log 25 - \log 3^2$

$$= \log\frac{25}{3^2}$$

(iii) $2\log x - 3\log y$

Sol: $2\log x - 3\log y = \log x^2 - \log y^3$

$$= \log\frac{x^2}{y^3}$$

(iv) $\log 5 + \log 6 - \log 2$

Sol: $\log 5 + \log 6 - \log 2 = \log\left(\frac{5\times6}{2}\right)$

Q#4) calculate the following:

(i). $\log_3 2 \times \log_2 81$

Sol: $\log_3 2 \times \log_2 81$

(using $\log_a n = \frac{\log_b n}{\log_b a}$)

$$\log_3 2 \times \log_2 81 = \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$$

$$= \frac{\log 3^4}{\log 3}$$

$$= \frac{4\ \log 3}{\log 3}$$

$$= 4$$

(i). $\log_5 3 \times \log_3 25$

Sol:). $\log_5 3 \times \log_3 25$

(using $\log_a n = \frac{\log_b n}{\log_b a}$)

$$\log_5 3 \times \log_3 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$$

$$= \frac{\log 5^2}{\log 5}$$

$$= \frac{2\ \log 5}{\log 5}$$

$$= 2$$

Q#5) If $\log 2 = 0.3010$, $\log 3 = 0.4171$, and $\log 5 = 0.6990$, then find the values of the following:

(i). $\log 32$

Sol: $\log 32 = \log 2^5 = 5\ \log 2 = 5\ (0.3010)$

$$= 1.5050$$

(ii). $\log 24$

Sol: $\log 24 = \log(2^3 \times 3) = \log 2^3 + \log 3$

$$= 3\ \log 2 + \log 3 = 3\ (0.3010) + (0.4171)$$

$$= 0.9030 + 0.4171 = 1.3801$$

(iii). $\log \sqrt{3\frac{1}{3}}$

Sol: $\log \sqrt{3\frac{1}{3}} = \log \sqrt{\frac{10}{3}} = \log\left(\frac{10}{3}\right)^{\frac{1}{2}}$

$$= \frac{1}{2}\log\frac{10}{3}$$

$$= \frac{1}{2}(\log 10 - \log 3)$$

$$= \frac{1}{2}(\log(2 \times 5) - \log 3)$$

$$= \frac{1}{2}(\log 2 + \log 5 - \log 3)$$

$$= \frac{1}{2}(0.3010 + 0.6990 - 0.4171)$$

$$= \frac{1}{2}(0.5229)$$

$$= 0.2615$$

(iv). $\log\frac{8}{3}$

Sol: $\log\frac{8}{3} = \log 8 - \log 3 = \log 2^3 - log 3$

$$= 3\ \log 2 - \log 3 = 3\ (0.3010) - 0.4171$$

$$= 0.9030 - 0.4171$$

$$= 0.4259$$

(v). $\log 30$

Sol: $\log 30 = \log(2 \times 5 \times 3) =$

$$= \log 2 + \log 5 + \log 3$$

$$= 0.3010 + 0.6990 + 0.4171$$

$$= 1.4771$$

Application of logarithm

Example:

Show that

$$7\log\frac{16}{15} + 5\log\frac{25}{24} + \log\frac{81}{80} = log2$$

Solution:

$$L.H.S = 7\,log\frac{16}{15} + 5\,log\frac{25}{24} + log\frac{81}{80}$$

$$= 7[log16 - log15] + 5[log25 - log24] + 3[log81 - log80]$$

$$= 7[log2^4 - log(3 \times 5)] + 5[log5^2 - log(2^3 \times 3)] + 3[log3^4 - log(2^4 \times 5)]$$

$$= 7[4log2 - log3 - log5] + 5[2log5 - 3log2 - log3] + 3[4log3 - 4log2 - log5]$$

$$= (28 - 15 - 12)log2 + (-7 - 5 + 12)log3 + (-7 + 10 - 3)log5$$

$$= log2 + 0 + 0 = log2 = R.H.S$$

Example

Evaluate $\sqrt[3]{\frac{0.079222 \times (18.99)^2}{(5.79)^4 \times 0.94744}}$

Solution:

Let $y = \sqrt[3]{\frac{0.079222 \times (18.99)^2}{(5.79)^4 \times 0.94744}}$

$$\left(\frac{0.079222 \times (18.99)^2}{(5.79)^4 \times 0.94744}\right)^{\frac{1}{3}}$$

$$logy = \frac{1}{3}\log\left(\frac{0.079222 \times (18.99)^2}{(5.79)^4 \times 0.94744}\right)$$

$$= \frac{1}{3}[\log\{0.07921 \times (18.99)\} - \log\{(5.79)^2 \times 0.9474\}]$$

$$= \frac{1}{3}[log0.07921 + 2log18.99 - 4log5.79 - log0.9474]$$

$$= \frac{1}{3}[\bar{2}.8988 + 2(1.2786) - 4(0.7627) - \bar{1}.9765]$$

$$= \frac{1}{3}[\bar{2}.8988 + 2.5572 - 3.0508 + 1 - 0.9765]$$

$$= \frac{1}{3}(\bar{2}.4287)$$

$$= \frac{1}{3}(\bar{3} + 1.4287)$$

$= 1 + 0.4762 = 1.4762$

$y = antilog\bar{1}.4762 = 0.299333$

Example:

Given $A = A_o e^{-kd}$ $if\ k = 2\ what\ should\ be\ the$ Value of d $to\ make\ A = \frac{A_0}{2}$?

Solution:

Given that $A = A_0 e^{-kd}$ $\Rightarrow \frac{A}{A_0} = e^{-kd}$

Subtracting $k = 2$ $and\ A = \frac{A_0}{2},\ we\ get \frac{1}{2} = e^{-2d}$

Taking common log on both sides

$$\log_{10} 1 - \log_{10} 2 = -2dlog_{10}e$$

Where $e = 2.718$

$$0 - 0.3010 = -2d(0.4343)$$

$$d = \frac{0.3010}{2 \times 0.4343} = 0.3465$$

Example 3.4

1. Using log tables to find the value of.

(i) 0.8176×13.64

Sol: Let $x = 0.8176 \times 13.64$

Taking log on both sides

$\log x = \log(0.8176 \times 13.64)$

$= \log 0.8176 + \log 13.64$

(In $\log 0.8176$, the ch. Is $\bar{1}$ we find the log(8.176) which is 0.9125, so combine both that is $\log 0.8176 = \bar{1} + 0.9125 = \bar{1}.9125$)

$= \bar{1}.9125 + 1.1348$

$= -1 + 0.9125 + 1.1348$

$= -0.0875 + 1.1348$

$\log x = 1.0473$

Taking anti-log on both sides, we have

$x = Antilog(1.0473)$

$x = 11.15$

(ii) $(789.5)^{\frac{1}{8}}$

Sol: Let $x = (789.5)^{\frac{1}{8}}$

Taking log on both sides

$logx = \log(789.5)^{\frac{1}{8}}$

$= \frac{1}{8}[\log(789.5)]$

$= \frac{1}{8}[2.8974] = \frac{2.8974}{8}$

$\log x = 0.3622$

Taking anti-log on both sides, we have

$x = Antilog(0.3622)$

$x = 2.302$

(iii) $\frac{0.678 \times 9.01}{0.0234}$

Sol: Let $x = \frac{0.678 \times 9.01}{0.0234}$

Taking log on both sides

$$\log x = \log\left(\frac{0.678 \times 9.01}{0.0234}\right)$$

$= \log(0.678) + \log(9.01) - \log(0.0234)$

$= \bar{1}.8312 + 0.9547 - \bar{2}.3692$

$= (-1 + 0.83120) + 0.9547 - (-2 + 0.3692)$

$= (-0.1688) + 0.9547 - (-1.6308)$

$= -0.1688 + 0.9547 + 1.6308$

$= 2.4163$

$\log x = 2.4163$

Taking anti-log on both side, we have

$x = Antilog(2.4163)$

$x = 261$

(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Sol: Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Taking log on both sides

$logx = \log(\sqrt[5]{2.709} \times \sqrt[7]{1.239})$

$= \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$

$= \frac{1}{5}[\log(2.709)] + \frac{1}{7}[\log(1.239)]$

$= \frac{1}{5}[0.4328] + \frac{1}{7}[0.1931]$

$= \frac{0.4328}{5} + \frac{0.1931}{7}$

$= 0.0866 + 0.0133$

$\log x = 0.0999$

Taking anti-log on both sides, we have

$x = Antilog(0.0999)$

$x = 1.258$

(v) $\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Sol: Let $x = \frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log on both sides

$$\log x = \log\left(\frac{(1.23)(0.6975)}{(0.0075)(1278)}\right)$$

$= \log(1.23) + \log(0.6975) - \log(0.0075) - \log(1279)$

$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$

$= 0.0899 + (-1 + 0.8435) - (-3 + 0.8751 - 3.1065$

$= 0.0899 + (-0.1565) - (-2.1249) - 3.1065$

$= 0.0899 - 0.1565 + 2.1249 - 3.1065$

$\log x = -1.0482$

Adding and subtracting 2 on R.H.S

$\log x = -2 + 2 - 1.0482$

$\log x = \bar{2} + (2 - 1.0482)$

$\log x = \bar{2} + (0.9518)$

$\log x = \bar{2}.9518$

Taking anti-log on both side, we have

$x = Antilog(\bar{2}.9518)$

(Here $Antilog(0.9518) = 8.50$ but Ch. $\bar{2}$ indicates that point will move two digits to left side)

$x = 0.0850$

(iii) $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

Sol: Let $x = \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

Taking log on both sides

$$\log x = \log\left(\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}\right)$$

$$= \log\left(\frac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$$

$= \frac{1}{3}[\log(0.7214) + \log(20.37) - \log(60.8)]$

$= \frac{1}{3}[\bar{1}.8582 + 1.3090 - 1.7839]$

$= \frac{1}{3}[-1 + 0.8582 + 1.3090 - 1.7839]$

$= \frac{1}{3}[-0.1418 + 1.3090 - 1.7839]$

$= \frac{1}{3}[-0.6157]$

$= -\frac{0.6157}{3}$

$\log x = -0.2056$

Adding and subtracting 1 on R.H.S

$\log x = -1 + 1 - 0.2056$

$\log x = \bar{1} + (1 - 0.2056)$

$\log x = \bar{1} + (0.7944)$

$\log x = \bar{1}.7944$

Taking anti-log on both side, we have

$x = Antilog(\bar{1}.7944)$

(Here $Antilog(0.7944) = 6.229$ but Ch. $\bar{1}$ indicates that point will move one digits to left side)

$x = 0.6229$

(v) $\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Sol: Let $x = \frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Taking log on both sides

$\log x = \log\left(\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}\right)$

$= \log(83) + \log(92)^{\frac{1}{3}} - \log(127) - \log(246)^{\frac{1}{5}}$

$= \log(83) + \frac{1}{3}\log(92) - \log(127) - \frac{1}{5}\log(246)$

$= 1.9191 + \frac{1}{3}(1.9638) - 2.1038 - \frac{1}{5}(2.3909)$

$= 1.9191 + \frac{1.9638}{3} - 2.1038 - \frac{2.3909}{5}$

$= 1.9191 + 0.6546 - 2.1038 - 0.4782$

$\log x = 1.9917$

Taking anti-log on both side, we have

$x = Antilog(1.9917)$

$x = 0.9811$

(viii) $\frac{(438)^3\sqrt{0.056}}{(388)^4}$

Sol: Let $x = \frac{(438)^3\sqrt{0.056}}{(388)^4}$

Taking log on both sides

$\log x = \log\left(\frac{(438)^3\sqrt{0.056}}{(388)^4}\right)$

$= \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$

$= 3\log(438) + \frac{1}{2}\log(0.056) - 4\log(388)$

$= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$

$= 7.9245 + \frac{1}{2}(-2 + 0.7482) - 10.3552$

$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$

$= 7.9245 - 0.6259 - 10.3552$

$\log x = -3.0566$

Adding and subtracting 4 on R.H.S

$\log x = -4 + 4 - 3.0566$

$\log x = \bar{4} + (4 - 3.0566)$

$\log x = \bar{4} + (0.9434)$

$\log x = \bar{4}.9434$

Taking anti-log on both side, we have

$x = Antilog(\bar{4}.9434)$

(Here $Antilog(0.9434) = 8.778$ but Ch. $\bar{4}$ indicates that point will move four digits to left side)

$x = 0.0008778$

Q#2) A gas is expanding according to the law $pv^n = C$. Find C, when $p = 80, v = 3.1$ and $n = \frac{5}{4}$.

Sol: $pv^n = C$

Taking log on both sides

$\log(pv^n) = \log C$

$\log C = \log p + \log v^n$

$\log C = \log p + n\log v$

Putting values

$\log C = \log 80 + \frac{5}{4}\log 3.1$

$\log C = 1.9030 + \frac{5}{4}(0.4914)$

$\log C = 1.9030 + 0.6143$

$\log C = 2.5173$

Taking anti-log on both side, we have

$C = Antilog(2.5173)$

$C = 329.2$

Q#3) The formula $p = 90\,(5)^{-\frac{q}{10}}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs. 18.00?

Sol: $p = 90\,(5)^{-\frac{q}{10}}$

Taking log on both sides

$\log(p) = \log(90\,(5)^{-\frac{q}{10}})$

$\log p = \log 90 + \log((5)^{-\frac{q}{10}})$

$\log p = \log 90 - \frac{q}{10}\log 5$

Putting values

$\log 18 = \log 90 - \frac{q}{10}\log 5$

$1.2553 = 1.9542 - \frac{q}{10}(0.6990)$

$1.2553 - 1.9542 = -\frac{q}{10}(0.6990)$

$-0.6990 = -\frac{q}{10}(0.6990)$

$1 = \frac{q}{10}$

$q = 10$ units

Q#4) If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$

Sol: $A = \pi r^2$

Taking log on both sides

$\log(A) = \log(\pi r^2)$

$$\log(A) = \log\left(\frac{22r^2}{7}\right)$$

$$\log A = \log 22 + \log(r)^2 - \log 7$$
$$\log A = \log 22 + 2\log r - \log 7$$

Putting values

$$\log A = \log 22 + 2\log 15 - \log 7$$
$$\log A = 1.3424 + 2(1.1761) - 0.8451$$
$$\log A = 1.3424 + 2.3522 - 0.8451$$
$$\log A = 2.8495$$

Taking anti-log on both side, we have

$$A = Antilog(2.8495)$$
$$A = 707.1 \text{ Sq. units}$$

Q#5) If $A = \frac{1}{3}\pi r^2 h$, find A, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$

Sol: $A = \frac{1}{3}\pi r^2 h$

Taking log on both sides

$$\log(A) = \log\left(\frac{1}{3}\pi r^2 h\right)$$
$$\log(A) = \log\left(\frac{22r^2 h}{21}\right)$$
$$\log A = \log 22 + \log(r)^2 + \log h - \log 21$$
$$\log A = \log 22 + 2\log r + \log h - \log 21$$

Putting values

$$\log A = \log 22 + 2\log 2.5 + \log 4.2 - \log 21$$
$$\log A = 1.3424 + 2(0.3979) + 0.6232 - 1.3222$$
$$\log A = 1.3424 + 0.7958 + 0.6232 - 1.3222$$
$$\log A = 1.4392$$

Taking anti-log on both side, we have

$$A = Antilog(1.4392)$$
$$A = 27.49 \text{ cubic units}$$

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MATHEMATICS 9th Science Group

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Chapter 4.

ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS

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Contents

Algebraic Expressions

Algebra is a generalization of arithmetic. Recall that when operations of addition and subtraction are applied to algebraic terms, we obtain an algebraic expression. For instance, $5x^2 - 3x + \frac{2}{\sqrt{x}}$, $3xy + \frac{3}{x}\ (x \neq 0)$ are algebraic expressions.

Polynomials

it is a polynomial A polynomial in the variable x is an algebraic expression of the form

$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x + a_0,\quad a_n \neq 0$ …… (i)

where n, the highest power of x, is a non-negative integer called the degree of the polynomial and each coefficient a_n, is a real number. The coefficient a_n of the highest power of x is called the *leading coefficient*

of the polynomial. $2x^4y^2 + x^2y^2 + 8x$ is a polynomial in two variables x and y having degree 6 (4+2=6).

Rational Expression

The quotient $\frac{p(x)}{q(x)}$ of two polynomials, $p(x)$ and $q(x)$, where $q(x)$

is a non-zero polynomial, is called a rational expression.

For example, $\frac{2x+5}{5x-1}$, $5x - \neq 0$ is a rational expression.

Note:

Every polynomial $p(x)$ can be regarded as a rational expression, since we can write $p(x)$ as $\frac{p(x)}{1}$.

Thus, every polynomial is a rational expression, but every rational expression need not be a polynomial.

Algebraic formulas

(i). $(a+b)^2 = a^2 + b^2 + 2ab$

(ii). $(a-b)^2 = a^2 + b^2 - 2ab$

(iii). $x^2 - y^2 = (x-y)(x+y)$

(iv) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(v) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

(vi) $\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$

(vii) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(viii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

EXERCISE4.1

Q#1) Identify whether the following algebraic expressions are polynomials (Yes or No).

(i). $3x^2 + \frac{1}{x} - 5$

Sol: No, it is not a polynomial because it contains the term $\frac{1}{x}$.

(ii). $3x^3 - 4x^2 - x\sqrt{x} + 3$

Sol: No, it is not a polynomial because it contains the term $x\sqrt{x}$.

(iii). $x^2 - 3x + \sqrt{2}$

Sol: Yes, it is a polynomial because all powers are non-negative integers.

(iv). $\frac{3x}{2x-1} + 8$

Sol: No, it is not a polynomial because it contains the term $\frac{3x}{2x-1}$.

Q#2) State whether each of the following expressions is a rational expression or not.

(i). $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Sol: It is not rational expression.

(ii). $\frac{x^3-2x^2+\sqrt{3}}{2+3x-x^2}$

Sol: It is not rational expression.

(iii). $\frac{x^2+6x+9}{x^2-9}$

Sol: It a rational expression.

(iv). $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Sol: It is not rational expression.

Q#3) Reduce the following rational expressions to the lowest form.

(i). $\frac{120\,x^2\,y^3\,z^5}{30\,x^3 y\,z^2}$

Sol: $\frac{120\,x^2\,y^3\,z^5}{30\,x^3 y\,z^2} = 4x^{2-3}\,y^{3-1}\,z^{5-2}$

$= 4x^{-1}\,y^2\,z^3$

$= \frac{4\,y^2\,z^3}{x}$

(ii). $\frac{8a\,(x+1)}{2(x^2-1)}$

Sol: $\frac{8a\,(x+1)}{2(x^2-1)} = \frac{4a\,(x+1)}{(x-1)(x+1)}$

$= \frac{4a}{(x-1)}$

$= \frac{4\,a}{x-1}$

(iii). $\frac{(x+y)^2-4xy}{(x-y)^2}$

Sol: $\frac{(x+y)^2-4xy}{(x-y)^2} = \frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy}$

$== \frac{x^2+y^2-2xy}{x^2+y^2-2xy} = 1$

(iv). $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

Sol: $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)} = \frac{(x^3-y^3)(x^2-2xy+y^2)}{(x^3-y^3)}$

$= (x-y)^2$

(v). $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

Sol: $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)} = \frac{(x+2)(x-1)(x+1)}{(x+1)(x^2-2^2)}$

$= \frac{(x+2)(x-1)}{(x-2)(x+2)} = \frac{x-1}{x-2}$

(vi). $\frac{x^2-4x+4}{2x^2-8}$

Sol: $\frac{x^2-4x+4}{2x^2-8} = \frac{x^2-2(x)(2)+(2)^2}{2(x^2-4)}$

$$= \frac{(x-2)^2}{2(x^2-2^2)}$$
$$= \frac{(x-2)^2}{2(x-2)(x+2)}$$
$$= \frac{x-2}{2(x+2)}$$

(vii). $\frac{64x^5-64x}{(8x^2+8)(2x+2)}$

Sol: $\frac{64x^5-64x}{(8x^2+8)(2x+2)} = \frac{64x(x^4-1)}{8(x^2+1)2(x+1)}$
$$= \frac{4x((x^2)^2-(1)^2)}{(x^2+1)(x+1)}$$
$$= \frac{4x(x^2+1)(x^2-1)}{(x^2+1)(x+1)}$$
$$= \frac{4x(x-1)(x+1)}{(x+1)}$$
$$= 4x(x-1)$$

(viii). $\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$

Sol: $\frac{9x^2-(x^2-4)^2}{4+3x-x^2} = \frac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$
$$= \frac{(3x-(x^2-4))(3x+(x^2-4))}{4+3x-x^2}$$
$$= \frac{(3x-x^2+4)(3x+x^2-4)}{4+3x-x^2}$$
$$= 3x+x^2-4$$

Q#4) Evaluate (a). $\frac{x^3y-2z}{xz}$ **for**

(i). $x=3, y=-1 \; and \; z=-2$

Sol: As given $\frac{x^3y-2z}{xz}$

Putt $x=3, y=-1 \; and \; z=-2$ **in above**

$$\frac{x^3y-2z}{xz} = \frac{(3)^3(-1)-2(-2)}{(3)(-2)}$$
$$= \frac{(27)(-1)+4}{-6}$$
$$= \frac{-27+4}{-6}$$
$$= \frac{-23}{-6} = \frac{23}{6} = 3\frac{5}{6}$$

(ii). $x=-1, y=-9 \; and \; z=4$

Sol: As given $\frac{x^3y-2z}{xz}$

Putt $x=-1, y=-9 \; and \; z=4$ **in above**

$$\frac{x^3y-2z}{xz} = \frac{(-1)^3(-9)-2(4)}{(-1)(4)}$$
$$= \frac{(-1)(-9)-8}{-4}$$
$$= \frac{+9-8}{-4}$$
$$= \frac{1}{-4} = -\frac{1}{4}$$

(b). $\frac{x^2y^3-5z^4}{xyz}$ **for** $x=4, y=-2 \; and \; z=-1$

Sol: As given $\frac{x^2y^3-5z^4}{xyz}$

Putt $x=4, y=-2 \; and \; z=-1$ **in above**

$$\frac{x^2y^3-5z^4}{xyz} = \frac{(4)^2(-2)^3-5(-1)^4}{(4)(-2)(-1)}$$
$$= \frac{(16)(-8)-5(1)}{8}$$
$$= \frac{-128-5}{8}$$
$$= \frac{-133}{8} = -16\frac{5}{8}$$

Q#5) Perform the indicated operation and simplify.

(i). $\frac{15}{2x-3y} - \frac{4}{3y-2x}$

Sol: $\frac{15}{2x-3y} - \frac{4}{3y-2x} = \frac{15}{2x-3y} - \frac{4}{-(2x-3y)}$
$$= \frac{15}{2x-3y} + \frac{4}{2x-3y}$$
$$= \frac{15+4}{2x-3y}$$
$$= \frac{19}{2x-3y}$$

(ii). $\frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$

Sol: $\frac{1+2x}{1-2x} - \frac{1-2x}{1+2x} = \frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$
$$= \frac{[(1)^2+(2x)^2+2(1)(2x)]-[(1)^2+(2x)^2-2(1)(2x)]}{(1)^2-(2x)^2}$$
$$= \frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$
$$= \frac{8}{1-4x^2}$$

(iii). $\frac{x^2-25}{x^2-36} - \frac{x+5}{x+6}$

Sol $\frac{x^2-25}{x^2-36} - \frac{x+5}{x+6} = \frac{x^2-5^2}{x^2-6^2} - \frac{x+5}{x+6}$
$$= \frac{(x+5)(x-5)}{(x+6)(x-6)} - \frac{x+5}{x+6}$$
$$= \frac{(x+5)(x-5)-(x-6)(x+5)}{(x+6)(x-6)}$$
$$= \frac{(x^2-25)-(x^2+5x-6x-30)}{x^2-36}$$
$$= \frac{(x^2-25)-(x^2-x-30)}{x^2-36}$$
$$= \frac{x^2-25-x^2+x+30}{x^2-36}$$
$$= \frac{x+5}{x^2-36}$$

(iv). $\frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2}$

Sol: $\frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2} = \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{(x-y)(x+y)}$
$$= \frac{x(x+y)-y(x-y)-2xy}{(x-y)(x+y)}$$
$$= \frac{x^2+xy-xy+y^2-2xy}{(x-y)(x+y)}$$
$$= \frac{(x-y)^2}{(x-y)(x+y)}$$
$$= \frac{x-y}{x+y}$$

(v). $\frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18}$

Sol: $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}=\frac{x-2}{(x)^2+2(x)(3)+(3^2)}-\frac{x+2}{2(x^2-9)}$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x^2-3^2)}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{2(x-2)(x-3)-(x+2)(x+3)}{2(x-3)(x+3)^2}$$

$$=\frac{2(x^2-3x-2x+6)-(x^2+3x+2x+6)}{2(x-3)(x+3)^2}$$

$$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x-3)(x+3)^2}$$

$$=\frac{2x^2-10x+12-x^2-5x-6}{2(x-3)(x+3)^2}$$

$$=\frac{x^2-15x+6}{2(x-3)(x+3)^2}$$

(iv). $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$

Sol: $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$

$$=\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{(x-1)(x+1)(x^2+1)}$$

$$=\frac{1(x+1)(x^2+1)-1(x-1)(x^2+1)-2(x+1)(x-1)-4}{(x-1)(x+1)(x^2+1)}$$

$$=\frac{(x^3+x+x^2+1)-(x^3+x-x^2-1)-2(x^2-1)-4}{x^4-1}$$

$$=\frac{x^3+x+x^2+1-x^3-x+x^2+1-2x^2+2-4}{x^4-1}$$

$$=\frac{2x^2+4-2x^2-4}{x^4-1}$$

$$=\frac{0}{x^4-1}=0$$

Q#6) Perform the indicated operation and simplify.

(i). $(x^2-49).\frac{5x+2}{x+7}$

Sol: $(x^2-49).\frac{5x+2}{x+7}=(x^2-7^2).\frac{5x+2}{x+7}$

$$=(x-7)(x+7).\frac{5x+2}{x+7}$$

$$=(x-7)(5x+2)$$

(ii). $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}$

Sol: $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}=\frac{4(x-3)}{x^2-3^2}\times\frac{(x)^2+2(x)(3)+(3^2)}{2(9-x^2)}$

$$=\frac{4(x-3)}{(x-3)(x+3)}\times\frac{(x+3)^2}{2(3-x)(3+x)}$$

$$=\frac{2}{1}\times\frac{1}{(3-x)}=\frac{2}{3-x}$$

(iii). $\frac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$

Sol: $\frac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$

$$=\frac{(x^2)^3-(y^2)^3}{(x-y)(x+y)}\times\frac{1}{(x^4+x^2y^2+y^4)}$$

$$=\frac{(x^2-y^2)((x^2)^2+(x^2)(y^2)+(y^2)^2)}{(x-y)(x+y)}\times\frac{1}{(x^4+x^2y^2+y^4)}$$

$$=\frac{(x-y)(x+y)(+x^2y^2+y^4)}{(x-y)(x+y)}\times\frac{1}{(x^4+x^2y^2+y^4)}$$

$$=1$$

(iv). $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$

Sol: $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}=\frac{(x-1)(x+1)}{(x+1)^2}.\frac{x+5}{1-x}$

$$=\frac{(x-1)}{(x+1)}.\frac{x+5}{-(x-1)}$$

$$=-\frac{(x+5)}{(x+1)}$$

(v). $\frac{x^2+xy}{y(x+y)}.\frac{x^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}$

Sol: $\frac{x^2+xy}{y(x+y)}.\frac{y^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}$

$$=\frac{x(x+y)}{y(x+y)}.\frac{x(x+y)}{y(x+y)}\times\frac{xy-2y}{x^2-x}$$

$$=\frac{x}{y}.\frac{x}{y}\times\frac{y(x-2)}{x(x-1)}$$

$$=\frac{x}{y}\times\frac{(x-2)}{(x-1)}$$

$$=\frac{x(x-2)}{y(x-1)}$$

Algebraic formulas

(i).$(a+b)^2+(a-b)^2=2(a^2+b^2)$

(ii).$(a+b)^2-(a-b)^2=4ab$

(iii). $(a+b+c)^2=a^2+b^2+c^2+2(ab+ac+ca)$

(iv) $(x+y)^3=x^3+y^3+3xy(x+y)$

(v) $(x-y)^3=x^3-y^3-3xy(x-y)$

(vi) $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

(vii) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(viii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

EXERCISE 4.2

Q#1).

(i) If $a+b=10$ and $a-b=6$, then find the value of a^2+b^2

Solution: As given $a+b=10$ and $a-b=6$

We find a^2+b^2=?

Using the identity

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Put values

$$(10)^2+(6)^2=2(a^2+b^2)$$

$$100+36=2(a^2+b^2)$$

$$136=2(a^2+b^2)$$

$$a^2+b^2=\frac{136}{2}$$

$$a^2+b^2=68$$

Which is required.

(ii) If $a+b=5$ and $a-b=\sqrt{17}$, then find the value of ab

Solution: As given $a+b=5$ and $a-b=\sqrt{17}$

We find ab=?

Using the identity

$$(a+b)^2-(a-b)^2=4ab$$
Put values
$$(5)^2-\left(\sqrt{17}\right)^2=4ab$$
$$25-17=4ab$$
$$8=4ab$$
$$ab=\frac{8}{4}$$
$$ab=2$$
Which is required.

Q#2) If $a^2+b^2+c^2=45$ and $a+b+c=-1$, then find the value of $ab+bc+ca$

Solution: As given $a^2+b^2+c^2=45$ and $a+b+c=-1$

We find $ab+bc+ca$=?

Using the identity

$(a+b+c)^2=a^2+b^2+c^2+2(ab+ac+ca)$

Put values
$$(-1)^2=45+2(ab+bc+ca)$$
$$1=45+2(ab+bc+ca)$$
$$1-45=2(ab+bc+ca)$$
$$-44=2(ab+bc+ca)$$
$$ab+bc+ca=-\frac{44}{2}$$
$$ab+bc+ca=-22$$
Which is required.

Q#3) If $m+n+p=10$ and $mn+np+mp=27$, then find the value of $m^2+n^2+p^2$

Solution: As given $m+n+p=10$ and $mn+np+mp=27$

We find $m^2+n^2+p^2$=?

Using the identity

$(m+n+p)^2=m^2+n^2+p^2+2(mn+np+mp)$

Put values
$$(10)^2=m^2+n^2+p^2+2(27)$$
$$100=m^2+n^2+p^2+54$$
$$100-54=m^2+n^2+p^2$$
$$m^2+n^2+p^2=46$$
Which is required.

Q#4) If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, then find the value of $x+y+z$

Solution: As given $x^2+y^2+z^2=78$ and $xy+yz+zx=59$

We find $x+y+z$ =?

Using the identity

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

Put values
$$(x+y+z)^2=78+2(59)$$
$$(x+y+z)^2=78+118$$
$$(x+y+z)^2=196$$
On taking square root, we get
$$\sqrt{(x+y+z)^2}=\pm\sqrt{196}$$
$$x+y+z=\pm 14$$
Which is required.

Q#5) If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, then find the value of $x+y+z$

Solution: As given $x+y+z=12$ and $x^2+y^2+z^2=64$

We find $xy+yz+zx$ =?

Using the identity

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

Put values
$$(12)^2=64+2(xy+yz+zx)$$
$$144=64+2(xy+yz+zx)$$
$$144-64=2(xy+yz+zx)$$
$$80=2(xy+yz+zx)$$
$$\frac{80}{2}=(xy+yz+zx)$$
$$xy+yz+zx=40$$
Which is required.

Q#6) If $x+y=7$ and $xy=12$, then find the value of x^3+y^3

Solution: As given $x+y=7$ and $xy=12$

We find x^3+y^3 =?

Using the identity
$$(x+y)^3=x^3+y^3+3xy(x+y)$$
Put values
$$(7)^3=x^3+y^3+3(12)(7)$$
$$343=x^3+y^3+252$$
$$343-252=x^3+y^3$$
$$x^3+y^3=91$$
Which is required.

Q#7) If $3x+4y=11$ and $xy=12$, then find the value of $27x^3+64y^3$

Solution: As given $3x+4y=11$ and $xy=12$

We find $27x^3+64y^3$ =?

Using the identity
$$(x+y)^3=x^3+y^3+3xy(x+y)$$
It becomes
$$(3x+4y)^3=(3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$$
$$(3x+4y)^3=27x^3+64y^3+36(xy)(3x+4y)$$
$$(11)^3=27x^3+64y^3+36(12)(11)$$
$$1331=27x^3+64y^3+4752$$
$$1331-4752=27x^3+64y^3$$
$$27x^3+64y^3=-3421$$

Q#8) If $x-y=4$ and $xy=21$, then find the value of x^3-y^3

Solution: As given $x-y=4$ and $xy=21$

We find x^3-y^3 =?

Using the identity

$(x-y)^3=x^3-y^3-3xy(x-y)$

Put values
$$(4)^3=x^3-y^3-3(21)(4)$$
$$64=x^3+y^3-252$$
$$64+252=x^3+y^3$$
$$x^3+y^3=316$$
Which is required value.

Q#9) If $5x-6y=13$ and $xy=6$, then find the value of $125x^3-216y^3$

Solution: As given $5x-6y=13$ and $xy=6$

We find $27x^3 + 64y^3 = ?$

Using the identity

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

It becomes

$$(5x-6y)^3 = (5x)^3 - (6y)^3 - 3(5x)(6y)(5x - 6y)$$

$$(5x-6y)^3 = 125x^3 - 216y^3 - 90(xy)(5x-6y)$$

$$(13)^3 = 125x^3 - 216y^3 - 90(6)(13)$$

$$2197 = 125x^3 - 216y^3 - 7020$$

$$2197 + 7020 = 125x^3 - 216y^3$$

$$125x^3 - 216y^3 = 9217$$

Which is required value.

Q#10) If $x + \frac{1}{x} = 3$ then find the value of $x^3 + \frac{1}{x^3}$

Solution: As given $x + \frac{1}{x} = 3$

We find $x^3 + \frac{1}{x^3} = ?$

Using the identity

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$$

Put values

$$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$27 - 9 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = 18$$

Which is required value.

Q#11) If $x - \frac{1}{x} = 7$ then find the value of $x^3 - \frac{1}{x^3}$

Sol: As given $x - \frac{1}{x} = 7$

We find $x^3 - \frac{1}{x^3} = ?$

Using the identity

$$\left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x-\frac{1}{x}\right)$$

Put values

$$(7)^3 = x^3 - \frac{1}{x^3} - 3(7)$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 364$$

Q#12) If $(3x + \frac{1}{3x}) = 5$ then find the value of $27x^3 + \frac{1}{27x^3}$

Solution: As given$3x + \frac{1}{3x} = 5$

We find $27x^3 + \frac{1}{27x^3} = ?$

Using the identity

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$$

It becomes

$$\left(3x+\frac{1}{3x}\right)^3 = (3x)^3 + \frac{1}{(3x)^3} + \left(3x+\frac{1}{3x}\right)$$

$$\left(3x+\frac{1}{3x}\right)^3 = 27x^3 + \frac{1}{27x^3} + \left(3x+\frac{1}{3x}\right)$$

$$(5)^3 = 27x^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

$$125 - 15 = 27x^3 + \frac{1}{27x^3}$$

$$27x^3 + \frac{1}{27x^3} = 110$$

Q#13) If $(5x - \frac{1}{5x}) = 6$ then find the value of $125x^3 - \frac{1}{125x^3}$

Solution: As given $5x - \frac{1}{5x} = 6$

We find $125x^3 - \frac{1}{125x^3} = ?$

Using the identity

$$\left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x-\frac{1}{x}\right)$$

It becomes

$$\left(5x-\frac{1}{5x}\right)^3 = (3x)^3 - \frac{1}{(3x)^3} - \left(5x-\frac{1}{5x}\right)$$

$$\left(5x-\frac{1}{5x}\right)^3 = 125x^3 - \frac{1}{125x^3} + \left(5x-\frac{1}{5x}\right)$$

$$(6)^3 = 125x^3 - \frac{1}{125x^3} - 3(6)$$

$$216 = 125x^3 - \frac{1}{125x^3} - 18$$

$$216 + 18 = 125x^3 - \frac{1}{125x^3}$$

$$125x^3 - \frac{1}{125x^3} = 234$$

Q#15)

(i). $x^3 - y^3 - x + y$

Sol: $x^3 - y^3 - x + y$

$= (x-y)(x^2+xy+y^2) - (x-y)$

$= (x-y)(x^2+xy+y^2-1)$

(ii). $8x^3 - \frac{1}{27y^3}$

Sol: $8x^3 - \frac{1}{27y^3}$

$= (2x)^3 - \left(\frac{1}{3y}\right)^3$

$= \left(2x - \frac{1}{3y}\right)\left((2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right)$

$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$

Q#16) Find the product, using formulas.

(i). $(x^2+y^2)(x^4-x^2y^2+y^4)$

Solution: $(x^2+y^2)(x^4-x^2y^2+y^4)$

$= (x^2+y^2)[(x^2)^2 - (x^2)(y^2) + (y^2)^2]$

Using identity

$x^3 + y^3 = (x+y)(x^2-xy+y^2)$

$= (x^2)^3 + (y^2)^3$

$= x^6 + y^6$

(ii). $(x^3-y^3)(x^6+x^3y^3+y^6)$

Sol: $(x^3-y^3)(x^6+x^3y^3+y^6)$

$= (x^3-y^3)[(x^3)^2+(x^3)(y^3)+(y^3)^2]$

Using identity

$x^3-y^3=(x-y)(x^2+xy+y^2)$

$=(x^3)^3-(y^3)^3$

$=x^9-y^9$

(iii). $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

Sol: $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

$=[(x-y)(x^2+xy+y^2)][(x+y)(x^2-xy+y^2)][(x^2+y^2)((x^2)^2-(x^2)(y^2)+(y^2)^2)]$

Using identity

$x^3+y^3=(x+y)(x^2-xy+y^2)$

$=[x^3-y^3][x^3+y^3][(x^2)^3+(y^2)^3]$

$=[(x^3)^2-(y^3)^2][x^6+y^6]$

$=(x^6-y^6)(x^6+y^6)$

$=(x^6)^2-(y^6)^2$

$=x^{12}-y^{12}$

(iv). $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$

Sol: $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$

$=[(2x^2-1)((2x^2)^2+(2x^2)(1)+(1)^2][(2x^2+1)((2x^2)^2-(2x^2)(1)+(1)^2]$

$=[(2x^2)^3-(1)^3][(2x^2)^3+(1)^3]$

$=(8x^6-1)(8x^6+1)$

$=(8x^6)^2-(1)^2$

$=64x^{12}-1$

Surd:

An irrational radical with rational radicand is called a surd.

That is $\sqrt[n]{a}$ surd if a is rational and $\sqrt[n]{a}$ is irrational.

For example, $\sqrt{2}, \sqrt[4]{5}, \sqrt{10}$

Also, $\sqrt{\pi}$ is not a surd because π is not rational.

$\sqrt{10+\sqrt{2}}$ is not a surd because $10+\sqrt{2}$ is not a rational number.

EXERCISE 4.3

1. Express each of the following surd in the simplest foam.

(i) $\sqrt{180}$

Solution: $\sqrt{180}=\sqrt{2\times2\times3\times3\times5}$

$=\sqrt{2^2\times3^2\times5}$

$=2\times3\sqrt{5}$

$=6\sqrt{5}$

(ii) $3\sqrt{162}$

Solution: $3\sqrt{162}=3\sqrt{2\times3\times3\times3\times3}$

$=3\sqrt{2\times3^2\times3^2}$

$=3\times3\times3\sqrt{2}$

$=27\sqrt{2}$

(iii) $\frac{3}{4}\sqrt[3]{128}$

Solution: $\frac{3}{4}\sqrt[3]{128}=\frac{3}{4}\sqrt[3]{2\times2\times2\times2\times2\times2\times2}$

$=\frac{3}{4}\sqrt[3]{2^3\times2^3\times2}$

$=\frac{3}{4}(2\times2\sqrt[3]{2})$

$=\frac{3}{4}(4\sqrt[3]{2})$

$=\mathbf{3\sqrt[3]{2}}$

(iv) $\sqrt[5]{96x^6y^7z^8}$

Solution: $\sqrt[5]{96x^6y^7z^8}$

$=\sqrt[5]{2\times2\times2\times2\times2\times3\times x^5\times y^5\times z^5\times x\times y^2\times z^3}$

$=\sqrt[5]{2^5\times x^5\times y^5\times z^5\times3\times x\times y^2\times z^3}$

$=2\times x\times y\times z\sqrt[5]{3\times x\times y^2\times z^3}$

$=2xyz\sqrt[5]{3xy^2z^3}$

Q#2) Simplify

(i). $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

Solution: $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}=\frac{\sqrt{2\times3\times3}}{\sqrt{3}\sqrt{2}}=\frac{\sqrt{2}\sqrt{3}\sqrt{3}}{\sqrt{3}\sqrt{2}}=\sqrt{3}$

(ii). $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

Solution: $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}=\frac{\sqrt{3\times7}\sqrt{3\times3}}{\sqrt{3\times3\times7}}=\frac{\sqrt{3}\sqrt{7}\sqrt{3}\sqrt{3}}{\sqrt{3}\sqrt{3}\sqrt{7}}=\sqrt{3}$

(iii). $\sqrt[5]{243x^5y^{10}z^{15}}$

Solution: $\sqrt[5]{243x^5y^{10}z^{15}}$

$=\sqrt[5]{3\times3\times3\times3\times3\times x^5\times y^5\times y^5\times z^5\times z^5\times z^5}$

$=\sqrt[5]{3^5\times x^5\times y^5\times y^5\times z^5\times z^5\times z^5}$

$=(3^5\times x^5\times y^5\times y^5\times z^5\times z^5\times z^5)^{\frac{1}{5}}$

$= 3^{5\times\frac{1}{5}} \times x^{5\times\frac{1}{5}} \times y^{5\times\frac{1}{5}} \times y^{5\times\frac{1}{5}} \times z^{5\times\frac{1}{5}} \times z^{5\times\frac{1}{5}} \times z^{5\times\frac{1}{5}}$
$= 3 \times x \times y \times y \times z \times z \times z$
$= 3xy^2z^3$

(iv) $\frac{4}{5}\sqrt[3]{125}$

Solution: $\frac{4}{5}\sqrt[3]{1258} = \frac{4}{5}\sqrt[3]{5\times5\times5}$
$= \frac{4}{5}\sqrt[3]{5^3}$
$= \frac{4}{5}(5)$
$= 4$

(v). $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

Solution: $\sqrt{21}\times\sqrt{7}\times\sqrt{3} = \sqrt{7\times3}\times\sqrt{7}\times\sqrt{3}$
$= \sqrt{7}\times\sqrt{3}\times\sqrt{7}\times\sqrt{3}$
$= (\sqrt{7})^2 \times (\sqrt{3})^2$
$= 7\times3 = 21$

Q#3) Simplify by combining similar terms.

(i). $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

Solution: $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = \sqrt{3\times3\times5} - 3\sqrt{2\times2\times5} + 4\sqrt{5}$
$= \sqrt{3^2\times5} - 3\sqrt{2^2\times5} + 4\sqrt{5}$
$= 3\sqrt{5} - 3\times2\sqrt{5} + 4\sqrt{5}$
$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$
$= \sqrt{5}(3-6+4)$
$= \sqrt{5}(1)$
$= \sqrt{5}$

(ii). $4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$

Solution: $4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$
$= 4\sqrt{2\times2\times3} + 5\sqrt{3\times3\times3} - 3\sqrt{5\times5\times3} + \sqrt{2\times2\times5\times5\times3}$
$= 4\sqrt{2^2\times3} + 5\sqrt{3^2\times3} - 3\sqrt{5^2\times3} + \sqrt{2^2\times5^2\times3}$
$= 4\times2\sqrt{3} + 5\times3\sqrt{3} - 3\times5\sqrt{3} + 2\times5\sqrt{3}$
$= 8\sqrt{5} + 15\sqrt{5} - 15\sqrt{5} + 10\sqrt{3}$
$= \sqrt{3}(8+15-15+10)$
$= \sqrt{5}(18)$
$= 18\sqrt{5}$

(iii). $\sqrt{3}(2\sqrt{3}+3\sqrt{3})$

Solution: $\sqrt{3}(2\sqrt{3}+3\sqrt{3}) = \sqrt{3}(5\sqrt{3})$
$= 5(\sqrt{3})^2 = 5(3) = 15$

(iv). $2(6\sqrt{5}-3\sqrt{5})$

Solution: $2(6\sqrt{5}-3\sqrt{5}) = 2(3\sqrt{5})$
$= 6\sqrt{5}$

Q#4) Simplify

(i). $(3+\sqrt{3})(3-\sqrt{3})$

Sol: $(3+\sqrt{3})(3-\sqrt{3})$
$= (3)^2 - (\sqrt{3})^2$
$= 9 - 3 = 6$

(ii). $(\sqrt{5}+\sqrt{3})^2$

Solution: $(\sqrt{5}+\sqrt{3})^2$
$= (\sqrt{5})^2 + (\sqrt{3})^2 + 2(\sqrt{5})(\sqrt{3})$
$= 5 + 3 + 2\sqrt{3\times5} = 8 + 2\sqrt{15}$

(iii). $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

Solution: $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$
$= (\sqrt{5})^2 - (\sqrt{3})^2$
$= 5 - 3 = 2$

(iv). $(\sqrt{2}+\frac{1}{\sqrt{3}})(\sqrt{2}-\frac{1}{\sqrt{3}})$

Solution: $(\sqrt{2}+\frac{1}{\sqrt{3}})(\sqrt{2}-\frac{1}{\sqrt{3}})$
$= (\sqrt{2})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$
$= 2 - \frac{1}{3} = \frac{6-1}{3} = \frac{5}{3}$

(i). $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

Solution: $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$
$= \left((\sqrt{x})^2 - (\sqrt{y})^2\right)(x+y)(x^2+y^2)$
$= (x-y)(x+y)(x^2+y^2)$
$= ((x)^2-(y)^2)(x^2+y^2)$
$= (x^2)^2 - (y^2)^2$
$= x^4 - y^4$

Surd:
An irrational radical with rational radicand is called a surd.
That is $\sqrt[n]{a}$ surd if a is rational and $\sqrt[n]{a}$ is irrational.
For example, $\sqrt{2}, \sqrt[4]{5}, \sqrt{10}$
Also, $\sqrt{\pi}$ is not a surd because π is not rational.
$\sqrt{10+\sqrt{2}}$ is not a surd because $10+\sqrt{2}$ is not a rational number.

Monomial surd:
A surd which contains a single term is called a monomial surd.
e.g., $\sqrt{2}, \sqrt{5}$ etc.

Binomial surd:
A surd which contains sum of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.
e.g., $\sqrt{2}+\sqrt{7}$ + or $\sqrt{12}-\sqrt{7}$ or $\sqrt{10}-\sqrt{2}$ etc.
We can extend this to the definition of a trinomial surd.

Rationalizing factor of the other
If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.

Rationalization
The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd.

Conjugate surd
Two binomial surds of second order differing only in sign connecting their terms are called conjugate surds.
Thus $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ are conjugate surds of each other.
The conjugate of $x+\sqrt{y}$ is $x-\sqrt{y}$.

The product of the conjugate surds $\sqrt{x}+\sqrt{y}$and $\sqrt{x}-\sqrt{y}$,

$$(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = (\sqrt{x})^2-(\sqrt{y})^2 = x-y$$

is a rational quantity independent of any radical.
Similarly, the product of $x+m\sqrt{y}$ nd its conjugate $x-m\sqrt{y}$ has

$$(x+m\sqrt{y})(x-m\sqrt{y}) = (x)^2-(m\sqrt{y})^2$$
$$= x^2-m^2y$$

and have no radical. For example,

$$(4+\sqrt{3})(4-\sqrt{3}) = (4)^2-(\sqrt{3})^2 = 16-3 = 13$$

, which is a rational number.

EXERCISE 4.4

1. Rationalize the denominator of the following.

(i) $\frac{3}{4\sqrt{3}}$

Sol: $\frac{3}{4\sqrt{3}} = \frac{3}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$$= \frac{3\sqrt{3}}{4(\sqrt{3})^2}$$
$$= \frac{3\sqrt{3}}{4\times 3}$$
$$= \frac{\sqrt{3}}{4}$$

(ii) $\frac{14}{\sqrt{98}}$

Solution: $\frac{14}{\sqrt{98}} = \frac{14}{\sqrt{98}}\times\frac{\sqrt{98}}{\sqrt{98}}$

$$= \frac{14\sqrt{98}}{(\sqrt{98})^2}$$
$$= \frac{14\sqrt{98}}{98}$$
$$= \frac{\sqrt{98}}{7}$$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

Solution: $\frac{6}{\sqrt{8}\sqrt{27}} = \frac{6}{\sqrt{216}} = \frac{6}{\sqrt{216}}\times\frac{\sqrt{216}}{\sqrt{216}}$

$$= \frac{6\sqrt{216}}{(\sqrt{216})^2}$$
$$= \frac{6\sqrt{6\times 6\times 6}}{216}$$
$$= \frac{6\times 6\sqrt{6}}{216}$$
$$= \frac{\sqrt{6}}{6}$$

(iv) $\frac{1}{3+2\sqrt{5}}$

Solution: $\frac{1}{3+2\sqrt{5}} = \frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$$= \frac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2}$$
$$= \frac{3-2\sqrt{5}}{9-(4\times 5)}$$
$$= \frac{3-2\sqrt{5}}{9-20}$$
$$= \frac{3-2\sqrt{5}}{-11}$$
$$= -\frac{1}{11}(3-2\sqrt{5})$$

(v) $\frac{15}{\sqrt{31}-4}$

Solution: $\frac{15}{\sqrt{31}-4} = \frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$

$$= \frac{15(\sqrt{31}+4)}{(\sqrt{31})^2-(4)^2}$$
$$= \frac{15(\sqrt{31}+4)}{31-16}$$
$$= \frac{15(\sqrt{31}+4)}{15}$$
$$= \sqrt{31}+4$$

(vi) $\frac{2}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{2}{\sqrt{5}-\sqrt{3}} = \frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$=\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$=\frac{2(\sqrt{5}+\sqrt{3})}{5-3}$$
$$=\frac{2(\sqrt{5}+\sqrt{3})}{2}$$
$$=\sqrt{5}+\sqrt{3}$$

(vi) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution: $\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$
$$=\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$
$$=\frac{(\sqrt{3})^2+(1)^2-2(\sqrt{3})(1)}{3-1}$$
$$=\frac{3+1-2\sqrt{3}}{2}$$
$$=\frac{4-2\sqrt{3}}{2}$$
$$=\frac{2(2-\sqrt{3})}{2}$$
$$=2-\sqrt{3}$$

(vi) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Sol: $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$
$$=\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$=\frac{(\sqrt{5})^2+(\sqrt{3})^2+2(\sqrt{5})(\sqrt{3})}{5-3}$$
$$=\frac{5+3+2\sqrt{15}}{2}$$
$$=\frac{8+2\sqrt{15}}{2}$$
$$=\frac{2(4+\sqrt{15})}{2}$$
$$=4+\sqrt{15}$$

Q#2) Find the conjugate of $x+\sqrt{y}$.

(i). $3+\sqrt{7}$

Solution: Let $z=3+\sqrt{7}$

Taking conjugate, we get
$$\bar{z}=\overline{3+\sqrt{7}}$$
$$\bar{z}=3-\sqrt{7}$$

(ii). $4-\sqrt{5}$

Solution: Let $z=4-\sqrt{5}$

Taking conjugate, we get
$$\bar{z}=\overline{4-\sqrt{5}}$$
$$\bar{z}=4+\sqrt{5}$$

(iii). $2+\sqrt{3}$

Solution: Let $z=2+\sqrt{3}$

Taking conjugate, we get
$$\bar{z}=\overline{2+\sqrt{3}}$$
$$\bar{z}=2-\sqrt{3}$$

(iv). $2+\sqrt{5}$

Solution: Let $z=2+\sqrt{5}$

Taking conjugate, we get
$$\bar{z}=\overline{2+\sqrt{5}}$$
$$\bar{z}=2-\sqrt{5}$$

(v). $5+\sqrt{7}$

Solution: Let $z=5+\sqrt{7}$

Taking conjugate, we get
$$\bar{z}=\overline{5+\sqrt{7}}$$
$$\bar{z}=5-\sqrt{7}$$

(vi). $4-\sqrt{15}$

Solution: Let $z=4-\sqrt{15}$

Taking conjugate, we get
$$\bar{z}=\overline{4-\sqrt{15}}$$
$$\bar{z}=4+\sqrt{15}$$

(vii). $7-\sqrt{6}$

Solution: Let $z=7-\sqrt{6}$

Taking conjugate, we get
$$\bar{z}=\overline{7-\sqrt{6}}$$
$$\bar{z}=7+\sqrt{6}$$

(viii). $9+\sqrt{2}$

Solution: Let $z=9+\sqrt{2}$

Taking conjugate, we get
$$\bar{z}=\overline{9+\sqrt{2}}$$
$$\bar{z}=9-\sqrt{2}$$

Q#3)

(i). If $x=2-\sqrt{3}$ find $\frac{1}{x}$

Solution: $x=2-\sqrt{3}$

And $\frac{1}{x}=\frac{1}{2-\sqrt{3}}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$
$$=\frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$
$$=\frac{2+\sqrt{3}}{4-3}$$
$$=\frac{2+\sqrt{3}}{1}$$
$$=2+\sqrt{3}$$

(ii). If $x=4-\sqrt{17}$ find $\frac{1}{x}$

Solution: $x=4-\sqrt{17}$

And $\frac{1}{x}=\frac{1}{4-\sqrt{17}}=\frac{1}{4-\sqrt{17}}\times\frac{4+\sqrt{17}}{4+\sqrt{17}}$
$$=\frac{4+\sqrt{17}}{(4)^2-(\sqrt{17})^2}$$
$$=\frac{4+\sqrt{17}}{16-17}$$
$$=\frac{4+\sqrt{17}}{-1}$$

$$= -(4+\sqrt{17})$$
$$= -4-\sqrt{17}$$

(iii). If $x = \sqrt{3}+2$ find $\frac{1}{x}$

Solution: $x = \sqrt{3}+2$

And $\frac{1}{x} = \frac{1}{\sqrt{3}+2} = \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

$$= \frac{\sqrt{3}-2}{(\sqrt{3})^2-(2)^2}$$
$$= \frac{\sqrt{3}-2}{3-4}$$
$$= \frac{\sqrt{3}-2}{-1}$$
$$= -(\sqrt{3}-2)$$
$$= -\sqrt{3}+2 = 2-\sqrt{3}$$

Q#4) Simplify

(vi) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Solution:

$$\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$
$$= \left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right) + \left(\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$$
$$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= \frac{[(1)(\sqrt{5})-(1)(\sqrt{3})+(\sqrt{2})(\sqrt{5})-(\sqrt{2})(\sqrt{3})]}{5-3} + \frac{[(1)(\sqrt{5})+(1)(\sqrt{3})-(\sqrt{2})(\sqrt{5})-(\sqrt{2})(\sqrt{3})]}{5-3}$$
$$= \frac{[\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}]}{2} + \frac{[\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}]}{2}$$
$$= \frac{(\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6})+(\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6})}{2}$$
$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$
$$= \frac{2\sqrt{5}-2\sqrt{6}}{2}$$
$$= \frac{2(\sqrt{5}-\sqrt{6})}{2}$$
$$= \sqrt{5}-\sqrt{6}$$

(ii) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$

Solution:

$$\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$$
$$= \left(\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}\right) + \left(\frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right) + \left(\frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$
$$= \frac{(2-\sqrt{3})}{(2)^2-(\sqrt{3})^2} + \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{(2-\sqrt{5})}{(2)^2-(\sqrt{5})^2}$$
$$= \frac{(2-\sqrt{3})}{4-3} + \frac{2(\sqrt{5}+\sqrt{3})}{5-3} + \frac{(2-\sqrt{5})}{4-5}$$
$$= \frac{(2-\sqrt{3})}{1} + \frac{2(\sqrt{5}+\sqrt{3})}{2} + \frac{(2-\sqrt{5})}{-1}$$
$$= (2-\sqrt{3})+(\sqrt{5}+\sqrt{3})-(2-\sqrt{5})$$
$$= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$
$$= 2\sqrt{5}$$

(iii) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$

Solution:

$$\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$$
$$= \left(\frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}+\sqrt{3}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) - \left(\frac{3}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right)$$
$$= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2} - \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$
$$= \frac{2(\sqrt{5}-\sqrt{3})}{5-3} + \frac{(\sqrt{3}-\sqrt{2})}{3-2} - \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$$
$$= \frac{2(\sqrt{5}-\sqrt{3})}{2} + \frac{(\sqrt{3}-\sqrt{2})}{1} - \frac{3(\sqrt{5}-\sqrt{2})}{3}$$
$$= (\sqrt{5}-\sqrt{3})+(\sqrt{3}-\sqrt{2})-(\sqrt{5}-\sqrt{2})$$
$$= \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2}$$
$$= 0$$

Q#5)

(i). If $x = 2+\sqrt{3}$ find $x-\frac{1}{x}$ and $\left(x-\frac{1}{x}\right)^2$

Solution: $x = 2+\sqrt{3}$

And $\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$
$$= \frac{2-\sqrt{3}}{4-3}$$
$$= \frac{2-\sqrt{3}}{1}$$
$$= 2-\sqrt{3}$$

Now, $x-\frac{1}{x} = (2+\sqrt{3})-(2-\sqrt{3})$
$$= 2+\sqrt{3}-2+\sqrt{3}$$
$$x-\frac{1}{x} = 2\sqrt{3}$$

Also, $\left(x-\frac{1}{x}\right)^2 = (2\sqrt{3})^2 = 4 \times 3 = 12$

(i). If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$ and $x^3+\frac{1}{x^3}$

Solution: $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

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$$= \frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$$
$$= \frac{(\sqrt{5})^2+(\sqrt{2})^2-2(\sqrt{5})(\sqrt{2})}{5-2}$$
$$= \frac{5+2-2\sqrt{10}}{3}$$
$$= \frac{1}{3}(7-2\sqrt{10})$$

And $\frac{1}{x} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$
$$= \frac{(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$$
$$= \frac{(\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})}{5-2}$$
$$= \frac{5+2+2\sqrt{10}}{3}$$
$$= \frac{1}{3}(7+2\sqrt{10})$$

Now, $x+\frac{1}{x} = \frac{1}{3}(7-2\sqrt{10}) + \frac{1}{3}(7+2\sqrt{10})$
$$= \frac{1}{3}(7-2\sqrt{10}+7+2\sqrt{10})$$
$$x+\frac{1}{x} = \frac{14}{3}$$

Using identity
$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

Putting values
$$\left(\frac{14}{3}\right)^2 = x^2+\frac{1}{x^2}+2$$
$$\frac{196}{9} = x^2+\frac{1}{x^2}+2$$
$$x^2+\frac{1}{x^2} = \frac{196}{9}-2$$
$$x^2+\frac{1}{x^2} = \frac{196-18}{9}$$
$$x^2+\frac{1}{x^2} = \frac{178}{9}$$

Also using the identity
$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

Putting values
$$\left(\frac{14}{3}\right)^3 = x^3+\frac{1}{x^3}+3\left(\frac{14}{3}\right)$$
$$\frac{2744}{27} = x^3+\frac{1}{x^3}+14$$
$$x^3+\frac{1}{x^3} = \frac{2744}{27}-14$$
$$x^3+\frac{1}{x^3} = \frac{2366}{27}$$

Q#6) Determine the rational numbers a and b if,
$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1} = a+\sqrt{3}b$

Solution:
$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1} = a+\sqrt{3}b$$
$$\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)+\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$
$$= a+\sqrt{3}b$$
$$\left(\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}\right)+\left(\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}\right) = a+\sqrt{3}b$$
$$\frac{(\sqrt{3})^2+(1)^2-2(\sqrt{3})(1)}{3-1}$$
$$+\frac{(\sqrt{3})^2+(1)^2+2(\sqrt{3})(1)}{3-1}$$
$$= a+\sqrt{3}b$$
$$\frac{3+1+2\sqrt{3}}{2}+\frac{3+1-2\sqrt{3}}{2} = a+\sqrt{3}b$$
$$\frac{4+2\sqrt{3}+4-2\sqrt{3}}{2} = a+\sqrt{3}b$$
$$\frac{8}{2} = a+\sqrt{3}b$$
$$4+0\sqrt{3} = a+\sqrt{3}b$$

On comparing, we get
$a = 4$ and $b = 0$

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MATHEMATICS 9th Science Group

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Chapter 5.

Factorization

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Contents

Introduction:

Factorization plays an important role in mathematics as it helps to reduce the study of complicated expressions to the study of simpler expressions. In this unit, we will deal different types of factorization of polynimials.

Factorization:

If a polynomial $p(x)$ can be expressed as $p(x) = g(x)h(x)$, then each of the polynomial $g(x) and\ h(x)$ is called a factor of $f(x)$.

Factorization of the expression of the type $ka + kb + kc$.

We will take common k from every term of the expression

$$ka + kb + kc = k(a + b + c)$$

Factorization of the type $ac + ad + bc + bd$

$$ac + ad + bc + bd = a(c + d) + b(c + d)$$
$$= (c + d)(a + b)$$

Factorization of the type $a^2 \pm 2ab + b^2$

$$(i).\quad a^2 + 2ab + b^2 = (a + b)^2$$
$$= (a + b)(a + b)$$
$$(ii).\quad a^2 - 2ab + b^2 = (a - b)^2$$
$$= (a - b)(a - b)$$

Factorization of the type $a^2 - b^2$

$$a^2 - b^2 = (a + b)(a - b)$$

Factorization of the type $a^2 \pm 2ab + b^2 - c^2$

$$a^2 \pm 2ab + b^2 - c^2 = (a \pm b)^2 - c^2$$
$$= (a \pm b + c)(a \pm b - c)$$

Exercise 5.1

Factorize.Question.1.

(i). $2abc - 4abx + 2abd$

Solution.

$$2abc - 4abx + 2abd = 2ab(c - 2x + d)$$

Answer.

(ii). $9xy - 12x^2y + 18y^2$

Solution.

$$9xy - 12x^2y + 18y^2 = 3y(3x - 4x^2 + 6y)$$

Answer.

(iii). $-3x^2y - 3x + 9xy^2$

Solution.

$$-3x^2y - 3x + 9xy^2 = -3x(y + 1 - 3y^2)$$

Answer.

(iv). $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

Solution.

$$5ab^2c^3 - 10a^2b^3c - 20a^3bc^2 = 5abc(bc^2 - 2ab^2 - 4a^2c)$$

Answer.

(v). $3x^3y(x - 3y) - 7x^2y^2(x - 3y)$

Solution.

$$3x^3\,y(x - 3y) - 7x^2\,y^2\,(x - 3y)$$
$$= x^2y(x - 3y)(3x - 7y)$$

Answer.

(vi). $2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)$

Solution.

$$2xy^3(x^2 + 5) + 8xy^2(x^2 + 5) = 2xy^2(x^2 + 5)(y + 4)$$

Answer.

Question.2.

(i). $5ax - 3ay - 5bx + 3by$

Solution.

$$5ax - 3ay - 5bx + 3by$$
$$= a(5x - 3y) - b(5x - 3y)$$
$$= (5x - 3y)(a - b)$$

Answer.

(ii). $3xy + 2y - 12x - 8$

Solution.

$$3xy + 2y - 12x - 8 = y(3x + 2) - 4(3x + 2)$$
$$= (3x + 2)(y - 4)$$

Answer.

(iii). $x^3 + 3xy^2 - 2x^2y - 6y^3$

Solution.

$$x^3 + 3xy^2 - 2x^2y - 6y^3$$
$$= x(x^2 + 3y^2) - 2y(x^2 + 3y^2)$$
$$= (x^2 + 3y^2)(x - 2y)$$

Answer.

(iv). $(x^2 - y^2)z + (y^2 - z^2)x$

Solution.

$$(x^2 - y^2)z + (y^2 - z^2)x$$
$$= x^2z - y^2z + y^2x - z^2x$$
$$= x^2z + y^2x - y^2z - z^2x$$
$$= x(xz + y^2) - z(y^2 + xz)$$
$$= (xz + y^2)(x - z)$$

Answer.

Question.3.

(i). $144a^2 + 24a + 1$

Solution.

$$144a^2 + 24a + 1 = (12a)^2 + 2(12a)(1) + (1)^2$$
$$= (12a + 1)^2$$

(ii). $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

Solution.

$$\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2} = \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2$$
$$= \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

(iii). $(x+y)^2-14z(x+y)+49z^2$

Solution.

$$(x+y)^2-14z(x+y)+49z^2$$
$$=(x+y)^2-2(x+y)(7z)+(7z)^2$$
$$=(x+y-7z)^2$$

(iv). $12x^2-36x+27$

Solution.

$$12x^2-36x+27=3(4x^2-12x+9)$$
$$=3[(2x)^2-2(2x)(3)+(3)^2]$$
$$=3(2x-3)^2$$

Question.4.

(i). $3x^2-75y^2$

Solution.

$$3x^2-75y^2=3(x^2-25y^2)$$
$$=3[(x)^2-(5y)^2]$$
$$=3(x+5y)(x-5y)$$

(ii). $x(x-1)-y(y-1)$

Solution.

$$x(x-1)-y(y-1)=x^2-x-y^2+y$$
$$=x^2-y^2-x+y$$
$$=(x+y)(x-y)-1(x-y)$$
$$=(x-y)(x+y-1)$$

(iii). $128am^2-242an^2$

Solution.

$$128am^2-242an^2=2a(64m^2-121n^2)$$
$$=2a[(8m)^2-(11n)^2]$$
$$=2a(8m+11n)(8m-11n)$$

(iv). $3x-243x^3$

Solution.

$$3x-243x^3=3x(1-81x^2)$$
$$=3x[(1)^2-(9x)^2]$$
$$=3x(1+9x)(1-9x)$$

Question.5.

(i). x^2-y^2-6y-9

Solution.

$$x^2-y^2-6y-9=x^2-(y^2+6y+9)$$
$$=x^2-[(y)^2+2(y)(3)+(3)^2]$$
$$=x^2-(y+3)^2$$
$$=[x+(y+3)][x-(y+3)]$$
$$=(x+y+3)(x-y-3)$$

(ii). x^2-a^2+2a-1

Solution.

$$x^2-a^2+2a-1=x^2-(a^2-2a+1)$$
$$=x^2-[(a)^2-2(a)(1)+(1)^2]$$
$$=(x)^2-(a-1)^2$$
$$=[x+(a-1)][x-(a-1)]$$
$$=(x+a-1)(x-a+1)$$

(iii). $4x^2-y^2-2y-1$

Solution.

$$4x^2-y^2-2y-1=4x^2-(y^2+2y+1)$$
$$=4x^2-[(y)^2+2(y)(1)+(1)^2]$$
$$=(2x)^2-(y+1)^2$$
$$=[2x+(y+1)][2x-(y+1)]$$
$$=(2x+y+1)(2x-y-1)$$

(iv). $x^2-y^2-4x-2y+3$

Solution.

$$x^2-y^2-4x-2y+3$$
$$=x^2-4x-y^2-2y+3$$
$$=x^2-4x+4-y^2-2y-1$$
$$=(x^2-4x+4)-(y^2+2y+1)$$
$$=[(x)^2-2(x)(2)+(2)^2]-[(y)^2+2(y)(1)+(1)^2]$$
$$=(x-2)^2-(y+1)^2$$
$$=[(x-2)+(y+1)][(x-2)-(y+1)]$$
$$=(x-2+y+1)(x-2-y-1)$$
$$=(x+y-1)(x-y-3)$$

(v). $25x^2-10x+1-36z^2$

Solution.

$$25x^2-10x+1-36z^2$$
$$=[(5x)^2-2(5x)(1)+(1)^2]-(6z)^2$$
$$=(5x-1)^2-(6z)^2$$
$$=(5x-1+6z)(5x-1-5z)$$
$$=(5x+6z-1)(5x-5z-1)$$

(vi). $x^2-y^2-4xz+4z^2$

Solution.

$$x^2-y^2-4xz+4z^2=x^2-4xz+4z^2-y^2$$
$$=(x)^2-2(x)(z)+(2z)^2-y^2$$
$$=(x-2z)^2-(y)^2$$
$$=(x-2z+y)(x-2z-y)$$
$$=(x+y-2z)(x-y-2z)$$

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(a) Factorization of the Expression of the types:

$a^4 + a^2b^2 + b^4$ or $a^4 + 4b^4$

Explanation: For $a^4 + a^2b^2 + b^4$

$$a^4 + a^2b^2 + b^4 = a^4 + b^4 + a^2b^2$$
$$= (a^2)^2 + (b^2)^2 + 2(a^2)(b^2) - 2(a^2)(b^2) + a^2b^2$$
$$= (a^2 + b^2)^2 - 2a^2b^2 + a^2b^2$$
$$= (a^2 + b^2)^2 - a^2b^2$$
$$= (a^2 + b^2)^2 - (ab)^2$$
$$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$$

Explanation: For $a^4 + 4b^4$

$$a^4 + 4b^4 = (a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2) - 2(a^2)(2b^2)$$
$$= (a^2 + 2b^2)^2 - 4a^2b^2$$
$$= (a^2 + 2b^2)^2 - (2ab)^2$$
$$= (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$$

(b) Factorization of the Expression of the types:

$x^2 + px + q$

Explanation:

$$x^2 + px + q = x^2 + (s + r)x + q\ ,$$
$$where\ s + r = p\ and\ s \times r = q$$
$$= x^2 + sx + rx + s \times r$$
$$= x(x + s) + r(x + s)$$
$$= (x + s)(x + r)$$

(c) Factorization of the Expression of the types:

$$ax^2 + bx + c, a \neq 0$$
$$ax^2 + bx + c = ax^2 + (s + r)x + c,$$
$$s + r = b\ and\ s \times r = ac$$
$$= ax^2 + sx + rx + \frac{s \times r}{a}$$
$$= x(ax + s) + r\left(x + \frac{s}{a}\right)$$
$$= x(ax + s) + r\left(\frac{ax + s}{a}\right)$$
$$= x(ax + s) + \frac{r}{a}(ax + s)$$
$$= (ax + s)\left(x + \frac{r}{a}\right)$$

(d) Factorization of the Expression of the types:

(i). $(ax^2 + bx + c)(ax^2 + bx + d) + k$

(ii). $(x + a)(x + b)(x + c)(x + d) + k$

(iii). $(x + a)(x + b)(x + c)(x + d) + kx^2$

Explanation: For $(ax^2 + bx + c)(ax^2 + bx + d) + k$

$$(ax^2 + bx + c)(ax^2 + bx + d) + k$$

We will suppose $ax^2 + bx = y$, *then above becomes*

$$= (y + c)(y + d) + k$$
$$= y^2 + yd + yc + k$$
$$= y^2 + (d + c)y + k$$

This the same type (b).

Explanation: For $(x + a)(x + b)(x + c)(x + d) + k$

$$(x + a)(x + b)(x + c)(x + d) + k$$

We will multiply the pair for which $a + b = c + d$, then

$$= [(x + a)(x + b)][(x + c)(x + d)] + k$$
$$= [x^2 + bx + ax + ab][x^2 + dx + cx + cd] + k$$
$$= (x^2 + (b + a)x + ab)(x^2 + (d + c)x + cd) + k$$

As $a + b = c + d$, *then*

$$= [x^2 + (c + d)x + ab][x^2 + (c + d)x + cd] + k$$

Suppose that

$$x^2 + (c + d)x = y\ , then\ above\ expression\ becomes$$
$$= (y + ab)(y + cd) + k$$
$$= y^2 + ycd + yab + abcd + k$$
$$= y^2 + (cd + ab)y + abcd + k$$

This the same type (b).

Explanation: For $(ax^2 + bx + c)(ax^2 + bx + d) + kx^2$

$$(x + a)(x + b)(x + c)(x + d) + kx^2$$

We will multiply the pair for which $a + b = c + d$, then

$$= [(x + a)(x + b)][(x + c)(x + d)] + kx^2$$
$$= [x^2 + bx + ax + ab][x^2 + dx + cx + cd] + kx^2$$
$$= (x^2 + (b + a)x + ab)(x^2 + (d + c)x + cd) + kx^2$$

As $a + b = c + d$, *then*

$$= [x^2 + (c + d)x + ab][x^2 + (c + d)x + cd] + kx^2$$

Suppose that

$$x^2 + (c + d)x = y\ , then\ above\ expression\ becomes$$
$$= (y + ab)(y + cd) + kx^2$$
$$= y^2 + ycd + yab + abcd + kx^2$$
$$= y^2 + (cd + ab)y + abcd + kx^2$$

After simplification it also becomes type (b).

(e). Factorization of the Expression of the type:

(i). $a^3 + 3a^2b + 3ab^2 + b^3$

(ii). $a^3 - 3a^2b + 3ab^2 - b^3$

Explanation For $a^3 + 3a^2b + 3ab^2 + b^3$

$$a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$$

It's a very famous formula.

Explanation For $a^3-3a^2b+3ab^2-b^3$

$$a^3-3a^2b+3ab^2-b^3=(a-b)^3$$

It's a very famous formula.

(f). Factorization of the Expression of the type:

(i). a^3+b^3

(ii). a^3-b^3

We will use, well known formulas for these types

(i). $a^3+b^3=(a+b)(a^2-ab+b^2)$

(ii). $a^3-b^3=(a-b)(a^2+ab+b^2)$

Exercise 5.2

Factorize

Question.1.

(i). $x^4+\frac{1}{x^4}-3$

Solution.

$$x^4+\frac{1}{x^4}-3=(x^2)^2+\left(\frac{1}{x^2}\right)^2-2(x^2)\left(\frac{1}{x^2}\right)+2(x^2)\left(\frac{1}{x^2}\right)-3$$
$$=\left(x^2-\frac{1}{x^2}\right)^2+2-3$$
$$=\left(x^2-\frac{1}{x^2}\right)^2-(1)^2$$
$$=(x^2-\frac{1}{x^2}+1)(x^2-\frac{1}{x^2}-1)$$

(ii). $3x^4+12y^4$

Solution.

$$3x^4+12y^4=3(x^4+4y^4)$$
$$=3[(x^2)^2+(2y^2)^2+2(x^2)(2y^2)-2(x^2)(2y^2)]$$
$$=3[(x^2+2y^2)^2-4x^2y^2]$$
$$=3[(x^2+2y^2)^2-(2xy)^2]$$
$$=3(x^2+2y^2+2xy)(x^2+2y^2-2xy)$$
$$=3(x^2+2xy+2y^2)(x^2-2xy+2y^2)$$

(iii). $a^4++3a^2b^2+4b^4$

Solution.

$$a^4++3a^2b^2+4b^4=a^4+4b^4+3a^2b^2$$
$$=(a^2)^2+(2b^2)^2+2(a^2)(2b^2)-2(a^2)(2b^2)+3a^2b^2$$
$$=(a^2+2b^2)^2-4a^2b^2+3a^2b^2$$
$$=(a^2+2b^2)^2-a^2b^2$$
$$=(a^2+2b^2)^2-(ab)^2$$
$$=(a^2+2b^2+ab)(a^2+2b^2-ab)$$

(iv). $4x^4+81$

Solution.

$$4x^4+81=(2x^2)^2+(9)^2+2(2x^2)(9)-2(2x^2)(9)$$
$$=(2x^2+9)^2-36x^2$$
$$=(2x^2+9)^2-(6x)^2$$
$$=(2x^2+9+6x)(2x^2+9-6x)$$
$$=(2x^2+6x+9)(2x^2-6x+9)$$

(v). x^4+x^2+25

Solution.

$$x^4+x^2+25=x^4+25+x^2$$
$$=(x^2)^2+(5)^2+2(x^2)(5)-2(x^2)(5)+x^2$$
$$=(x^2+5)^2-10x^2+x^2$$
$$=(x^2+5)^2-9x^2$$
$$=(x^2+5)^2-(3x)^2$$
$$=(x^2+5+3x)(x^2+5-3x)$$
$$=(x^2+3x+5)(x^2-3x+5)$$

(v). x^4+4x^2+16

Solution.

$$x^4+4x^2+16=x^4+16+4x^2$$
$$=(x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$$
$$=(x^2+4)^2-8x^2+4x^2$$
$$=(x^2+4)^2-4x^2$$
$$=(x^2+4)^2-(2x)^2$$
$$=(x^2+4+2x)(x^2+4-2x)$$
$$=(x^2+2x+4)(x^2-2x+4)$$

Question.2.

(i). $x^2+14x+48$

Solution.

$$x^2+14x+48=x^2+8x+6x+48$$
$$=x(x+8)+6(x+8)$$
$$=(x+8)(x+6)$$

(ii). $x^2-21x+108$

Solution.

$$x^2-21x+108=x^2-12x-9x+108$$
$$=x(x-12)-9(x-12)$$
$$=(x-12)(x-9)$$

(iii). $x^2-11x-42$

Solution.

$$x^2-11x-42=x^2-14x+3x-42$$
$$=x(x-14)+3(x-14)$$
$$=(x-14)(x+3)$$

(iii). $x^2+x-132$

Solution.

$$x^2+x-132=x^2+12x-11x-132$$
$$=x(x+12)-11(x+12)$$
$$=(x+12)(x-11)$$

Question.3.

(i). $4x^2+12x+5$

Solution.

$$4x^2+12x+5=4x^2+10x+2x+5$$
$$=2x(2x+5)+1(2x+5)$$
$$=(2x+5)(2x+1)$$

(ii). $30x^2+7x-15$

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Solution.

$$30x^2+7x-15=30x^2+25x-18x-15$$
$$=5x(6x+5)-3(6x+5)$$
$$=(6x+5)(5x-3)$$

(iii). $24x^2-65x+21$

Solution.

$$24x^2-65x+21=24x^2-56x-9x+21$$
$$=8x(3x-7)-3(3x-7)$$
$$=(3x-7)(8x-3)$$

(iv). $5x^2-16x-21$

Solution.

$$5x^2-16x-21=5x^2-21x+5x-21$$
$$=x(5x-21)+1(5x-21)$$
$$=(5x-21)(x+1)$$

(v). $4x^2-17xy+4y^2$

Solution.

$$4x^2-17xy+4y^2=4x^2-16xy-xy+4y^2$$
$$=4x(x-4y)-y(x-4y)$$
$$=(x-4y)(4x-y)$$

(vi). $3x^2-38xy-13y^2$

Solution.

$$3x^2-38xy-13y^2$$
$$=3x^2-39xy+xy-13y^2$$
$$=3x(x-13y)+y(x-13y)$$
$$=(x-13y)(3x+y)$$

(vii). $5x^2+33xy-14y^2$

Solution.

$$5x^2+33xy-14y^2$$
$$=5x^2+35xy-2xy-14y^2$$
$$=5x(x+7y)-2y(x+7y)$$
$$=(x+7y)(5x-2y)$$

(viii). $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4\,,x\neq 0.$

Solution.

$$\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$$

Suppose that $5x-\frac{1}{x}=y$

$$\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4=y^2+4y+4$$
$$=y^2+2y+2y+4$$
$$=y(y+2)+2(y+2)$$
$$=(y+2)(y+2)$$
$$=(y+2)^2$$
$$=\left(5x-\frac{1}{x}+2\right)^2$$

Question.4

(i). $(x^2+5x+4)(x^2+5x+6)-3$

Solution.

$$(x^2+5x+4)(x^2+5x+6)-3$$

Suppose that $x^2+5x=y$

$$=(y+4)(y+6)-3$$
$$=y^2+6y+4y+24-3$$
$$=y^2+10y+21$$
$$=y^2+7y+3y+21$$
$$=y(y+7)+3(y+7)$$
$$=(y+7)(y+3)$$
$$=(x^2+5x+7)(x^2+5x+3)$$

(ii). $(x^2-4x)(x^2-4x-1)-20$

Solution.

$$(x^2-4x)(x^2-4x-1)-20$$

Suppose that $x^2-4x=y$

$$=(y)(y-1)-20$$
$$=y^2-y-20$$
$$=y^2-5y+4y-20$$
$$=y(y-5)+4(y-5)$$
$$=(y-5)(y+4)$$
$$=(x^2-4x-5)(x^2-4x+4)$$
$$=(x^2-5x+x-5)(x^2-2x-2x+4)$$
$$=[x(x-5)+1(x-5)][x(x-2)-2(x-2)]$$
$$=(x-5)(x+1)(x-2)(x-2)$$
$$=(x-5)(x+1)(x-2)^2$$

(iii). $(x+2)(x+3)(x+4)(x+5)-15$

Solution.

$$(x+2)(x+3)(x+4)(x+5)-15$$
$$=(x+2)(x+5)(x+3)(x+4)-15$$
$$=(x^2+5x+2x+10)(x^2+4x+3x+12)-15$$
$$=(x^2+7x+10)(x^2+7x+12)-15$$

Suppose that $x^2+7x=y$

$$=(y+10)(y+12)-15$$
$$=y^2+12y+10y+120-15$$
$$=y^2+22y+105$$
$$=y^2+15y+7y+105$$
$$=y(y+15)+7(y+15)$$
$$=(y+15)(y+7)$$
$$(x^2+7x+15)(x^2+7x+7)$$

(iv). $(x+4)(x-5)(x+6)(x-7)-504$

Solution.

$$(x+4)(x-5)(x+6)(x-7)-504$$
$$(x+2)(x+5)(x+3)(x+4)-1$$
$$=(x+4)(x-5)(x+6)(x-7)-504$$
$$=(x^2-5x+4x-20)(x^2-7x+6x-42)-504$$
$$=(x^2-x-20)(x^2-x-42)-504$$

Suppose that $x^2-x=y$

$$=(y-20)(y-42)-504$$
$$=y^2-42y-20y+840-504$$
$$=y^2-62y+336$$

$$= y^2 - 56y - 6y + 336$$
$$= y(y-56) - 6(y-56)$$
$$= (y-56)(y-6)$$
$$= (x^2 - x - 56)(x^2 - x - 6)$$
$$= (x^2 - 8x + 7x - 56)(x^2 - 3x + 2x - 6)$$
$$[x(x-8) + 7(x-9)][x(x-3) + 2(x-3)]$$
$$= (x-9)(x+7)(x-3)(x+2)$$

(v). $(x+1)(x+2)(x+3)(x+6) - 3x^2$

Solution.

$$(x+1)(x+2)(x+3)(x+6) - 3x^2$$
$$= (x+1)(x+6)(x+2)(x+3) - 3x^2$$
$$= (x^2 + 6x + x + 6)(x^2 + 3x + 2x + 6) - 3x^2$$
$$= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$$
$$= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$$

Suppose that $x^2 + 6 = y$

$$= (y + 7x)(y + 5x) - 3x^2$$
$$= y^2 + 5xy + 7xy + 35x^2 - 3x^2$$
$$= y^2 + 12xy + 32x^2$$
$$= y^2 + 8xy + 4xy + 32x^2$$
$$= y(y + 8x) + 4x(y + 8x)$$
$$= (y + 8x)(y + 4x)$$
$$= (x^2 + 6 + 8x)(x^2 + 6 + 4x)$$
$$= (x^2 + 8x + 6)(x^2 + 4x + 6)$$

Question.5.

(i). $x^3 + 48x - 12x^2 - 64$

Solution.

$$x^3 + 48x - 12x^2 - 64$$
$$= x^3 - 12x^2 + 48x - 64$$
$$= (x)^3 - 3(x)^2(4) + 3(x)(4)^2 - (4)^3$$
$$= (x-4)^3$$

(ii). $8x^3 + 60x^2 + 150x + 125$

Solution.

$$8x^3 + 60x^2 + 150x + 125$$
$$= 8x^3 + 60x^2 + 150x + 5^3$$
$$= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$$
$$= (2x+5)^3$$

(iii). $x^3 - 18x^2 + 108x - 216$

Solution.

$$x^3 - 18x^2 + 108x - 216$$
$$= x^3 - 18x^2 + 108x - 6^3$$
$$= (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - (6)^3$$
$$= (x-6)^3$$

(iv). $8x^3 - 125y^3 - 60x^2y + 150xy^2$

Solution.

$$8x^3 - 125y^3 - 60x^2y + 150xy^2$$
$$= 8x^3 - 60x^2y + 150xy^2 - 125y^3$$
$$= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3$$
$$= (2x - 5y)^3$$

Question.6.

(i). $27 + 8x^3$

Solution.

$$27 + 8x^3 = (3)^3 + (2x)^3$$
$$= (3 + 2x)[(3)^2 - (3)(2x) + (2x)^2]$$
$$= (3 + 2x)(9 - 6x + 4x^2)$$
$$= (2x + 3)(4x^2 - 6x + 9)$$

(ii). $125x^3 - 216y^3$

Solution.

$$125x^3 - 216y^3 = (5x)^3 - (6y)^3$$
$$= (5x - 6y)[(5x)^2 + (5x)(6y) + (6y)^2]$$
$$= (5x - 6y)(25x^2 + 30xy + 36y^2)$$

(iii). $64x^3 + 27y^3$

Solution.

$$64x^3 + 27y^3 = (4x)^3 + (3y)^3$$
$$= (4x + 3y)[(4x)^2 - (4x)(3y) + (3y)^2]$$
$$= (4x + 3y)(16x^2 - 12xy + 9y^2)$$

(iv). $8x^3 + 125y^3$

Solution.

$$8x^3 + 125y^3 = (2x)^3 + (5y)^3$$
$$= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2]$$
$$= (2x + 5y)(4x^2 - 10xy + 25y^2)$$

Remainder Theorem and Factor Theorem:

Remainder Theorem:

If a polynomial $p(x)$ is divided by a linear divisor $(x - a)$, then the remainder is $p(a)$.

Zero of a Polynomial:

If a specific number $x = a$ is substituted for the variable x in a polynomial $p(x)$ so that the value $p(a)$ is zero, then $x = a$ is called a zero of the polynomial $p(x)$.

Factor Theorem:

The polynomial $(x - a)$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$.

Exercise 5.3

Question.1. Use the remainder theorem to find the remainder when

(i). $\boldsymbol{3x^3 - 10x^2 + 13x - 6}$ ***is divided by*** $\boldsymbol{(x-2)}$

Solution.

Suppose that $\boldsymbol{p(x) = 3x^3 - 10x^2 + 13x - 6}$ ***and***

$$\boldsymbol{x - 2 = 0}$$
$$\boldsymbol{x = 2}$$

Then

$$\textbf{Remainder} = p(2)$$
$$= 3(2)^3 - 10(2)^2 + 13(2) - 6$$
$$Remainder = 3(8) - 10(4) + 26 - 6$$
$$= 24 - 40 + 20$$
$$= 44 - 40$$
$$= 4$$

Hence required Remainder is 4.

(ii). $4x^3 - 4x + 3$ *is divided by* $(2x - 1)$

Solution.

Suppose that $p(x) = 4x^3 - 4x + 3$ *and*

$$2x - 1 = 0$$
$$2x = 1$$
$$x = \frac{1}{2}$$

Then

$$\textbf{Remainder} = p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3$$
$$Remainder = 4\left(\frac{1}{8}\right) - 2 + 3$$
$$= \frac{1}{2} + 1$$
$$= \frac{1 + 2}{2}$$
$$= \frac{3}{2}$$

Hence required Remainder is $\frac{3}{2}$.

(iii). $6x^4 + 2x^3 - x + 2$ *is divided by* $(x + 2)$

Solution.

Suppose that $p(x) = 6x^4 + 2x^3 - x + 2$ *and*

$$x + 2 = 0$$
$$x = -2$$

Then

$$\textbf{Remainder} = p(-2)$$
$$= 6(-2)^4 + 2(-2)^3 - (-2) + 2$$
$$Remainder = 6(16) + 2(-8) + 2 + 2$$
$$= 96 - 16 + 4$$
$$= 100 - 16$$
$$= 84$$

Hence required Remainder is 84.

(iv). $(2x - 1)^3 + 6(3 + 4x)^2 - 10$ *is divided by* $(2x + 1)$

Solution.

Suppose that $p(x) = (2x - 1)^3 + 6(3 + 4x)^2 - 10$ *and*

$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

Then

$$\textbf{Remainder} = p\left(-\frac{1}{2}\right)$$
$$= \left(2\left(-\frac{1}{2}\right) - 1\right)^3 + 6\left(3 + 4\left(-\frac{1}{2}\right)\right)^2 - 10$$
$$Remainder = (-1 - 1)^3 + 6(3 - 2)^2 - 10$$
$$= (-2)^3 + 6(1)^2 - 10$$
$$= -8 + 6 - 10$$
$$= 6 - 18$$
$$= -12$$

Hence required Remainder is -12.

(v). $x^3 - 3x^2 + 4x - 14$ *is divided by* $(x + 2)$

Solution.

Suppose that $p(x) = x^3 - 3x^2 + 4x - 14$ *and*

$$x + 2 = 0$$
$$x = -2$$

Then

$$\textbf{Remainder} = p(-2)$$
$$= (-2)^3 - 3(-2)^2 + 4(-2) - 14$$
$$Remainder = -8 - 3(4) - 8 - 14$$
$$= -8 - 12 - 8 - 14$$
$$= -42$$

Hence required Remainder is -42.

Question.2.

(i). If $(x + 2)$ is a factor of $x^2 - 4kx - 4k^2$, then find the value(s) of k.

Solution.

Suppose

$$p(x) = 3x^2 - 4kx - 4k^2$$

And $\quad x + 2 = 0$

$$x = -2.$$

Since $(x + 2)$ is factor of the polynomial $p(x)$, So

$$p(a) = 0$$
$$3(-2)^2 - 4k(-2) - 4k^2 = 0$$
$$3(4) + 8k - 4k^2 = 0$$
$$12 + 8k - 4k^2 = 0$$
$$4k^2 - 8k - 12 = 0$$
$$4(k^2 - 2k - 3) = 0$$
$$k^2 - 2k - 3 = 0$$
$$k^2 - 3k + k - 3 = 0$$
$$k(k - 3) + 1(k - 3) = 0$$
$$(k - 3)(k + 1) = 0$$
$$k - 3 = 0\ , \qquad k + 1 = 0$$
$$k = 3\ , \quad k = -1.$$

(ii). If $(x - 1)$ is a factor of $x^3 - kx^2 + 11x - 6$, then find the value(s) of k.

Solution.

Suppose

$$p(x) = x^3 - kx^2 + 11x - 6$$

And $x - 1 = 0$

$$x = 1.$$

Since $(x-1)$ is factor of the polynomial $p(x)$, So

$$p(1) = 0$$
$$(1)^3 - k(1)^2 + 11(1) - 6 = 0$$
$$1 - k + 11 - 6 = 0$$
$$6 - k = 0$$
$$k = 6.$$

Question.3.

Without actual long division determine whether (i). $(x-2)\,and\,(x-3)$ are factors of $p(x) = x^3 - 12x^2 + 44x - 48$.

Solution.

Suppose that

$$p(x) = x^3 - 12x^2 + 44x - 48$$

And $x - 2 = 0$, $x - 3 = 0$

$x = 2$, $x = 3$

Remainder for $x - 2\ is$

$$p(2) = (2)^3 - 12(2)^2 + 44(2) - 48$$
$$= 8 - 12(4) + 88 - 48$$
$$= 8 - 48 + 88 - 48$$
$$= 96 - 96$$
$$= 0$$

Hence $(x-2) is\ the\ factor\ of\ p(x)$.

Remainder for $x - 3\ is$

$$p(3) = (3)^3 - 12(3)^2 + 44(3) - 48$$
$$= 27 - 12(9) + 132 - 48$$
$$= 27 - 108 + 132 - 48$$
$$= 159 - 156$$
$$= 3 \neq 0$$

Hence $(x-3) is\ not\ factor\ of\ p(x)$.

Question.4.

For what value of m is the polynomial $p(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $(x+2)$?

Solution.

Suppose that

$$p(x) = 4x^3 - 7x^2 + 6x - 3m$$

And $x + 2 = 0$

$$x = -2.$$

Remainder for $x + 2\ is$

$$p(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$$
$$= 4(-8) - 7(4) - 12 - 3m$$
$$= -32 - 28 - 12 - 3m$$
$$= -72 - 3m$$

For the given condition $p(-2) = 0$

$$-72 - 3m = 0$$
$$-3m = 72$$
$$m = -\frac{72}{3} = -24.$$

Question.5.

Determine the value of k if $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$.

Solution.

Let $p(x) = kx^3 + 4x^2 + 3x - 4$.

By remainder theorem, $p(x)$ is divided by $(x-3)$, then remainder is

$$p(3) = k(3)^3 + 4(3)^2 + 3(3) - 4$$
$$p(3) = k(27) + 4(9) + 9 - 4$$
$$p(3) = 27k + 36 + 5$$
$$p(3) = 27k + 41$$

Also by remainder theorem, $q(x)$ is divided by $(x-3)$, then remainder is

$$q(3) = (3)^3 - 4(3) + k$$
$$q(3) = 27 - 12 + k$$
$$q(3) = 15 + k$$

By Given Condition, we have

$$p(3) = q(3)$$
$$27k + 41 = 15 + k$$
$$27k - k = 15 - 41$$
$$26k = -26$$
$$k = -1.$$

Question.6.

The remainder after dividing the polynomial $p(x) = x^3 + ax^2 + 7$ by $(x+1)$ is $2b$. Calculate the value of $a\ and\ b\ if\ this$ expression leaves a remainder of $(b+5)$ on being divided by $(x-2)$.

Solution.

Let $p(x) = x^3 + ax^2 + 7\ by\ (x+1)$.

By remainder theorem, $p(x)$ is divided by $(x+1)$, then remainder is 2b

$$p(-1) = 2b$$
$$(-1)^3 + a(-1)^2 + 7 = 2b$$
$$-1 + a + 7 = 2b$$
$$a + 6 = 2b$$
$$a = 2b - 6 - - - (i)$$

By remainder theorem, $p(x)$ is divided by $(x-2)$, then remainder is $b + 5$

$$p(2) = b + 5$$
$$(2)^3 + a(2)^2 + 7 = b + 5$$
$$8 + a(4) + 7 = b + 5$$
$$4a + 15 = b + 5$$
$$4a = b - 10 - - - (ii)$$

Using (i) in (ii), we have

$$4(2b - 6) = b - 10$$
$$8b - 24 = b - 10$$

$$8b - b = -10 + 24$$
$$7b = 14$$
$$b = \frac{14}{7}$$
$$b = 2$$

Using $b = 2$ in $(ii), we$ $have$

$$4a = 2 - 10$$
$$4a = -8$$
$$a = -\frac{8}{4}$$
$$a = -2$$

Hence $a = -2$ and $b = 2$.

Question.7.

The polynomial $x^3 + lx^2 + mx + 24$ has a factor $(x + 4)$ and it leaves a remainder of 36 when divided by $(x - 2)$. Find the values of l and m.

Solution.

Let $p(x) = x^3 + lx^2 + mx + 24$.

By remainder theorem, $p(x)$ is divided by $(x + 4)$, then remainder is 0.

$$p(-4) = 0$$
$$(-4)^3 + l(-4)^2 + m(-4) + 24 = 0$$
$$-64 + 16l - 4m + 24 = 0$$
$$16l - 4m - 40 = 0$$
$$4(4l - m - 10) = 0$$
$$4l - m - 10 = 0$$
$$4l - m = 10 - - - (i).$$

By remainder theorem, $p(x)$ is divided by $(x - 2)$, then remainder is 36.

$$p(2) = 36$$
$$(2)^3 + l(2)^2 + m(2) + 24 = 36$$
$$8 + 4l + 2m + 24 = 36$$
$$4l + 2m + 32 = 36$$
$$4l + 2m = 36 - 32$$
$$4l + 2m = 4$$
$$2(2l + m) = 4$$
$$2l + m = \frac{4}{2}$$
$$2l + m = 2 - - - (ii)$$

Using (i) in (ii), we have

$$4l - m = 10$$
$$2l + m = 2$$

$$6l = 12$$
$$l = \frac{12}{6}$$
$$l = 2.$$

Using $l = 2$ in $(ii), we$ $have$

$$2(2) + m = 2$$
$$4 + m = 2$$
$$m = 2 - 4$$
$$m = -2.$$

Hence $l = 2$ and $m = -2$.

Question.8.

The expression $lx^3 + mx^2 - 4$ $leaves$ remainder -3 and 12 $respectively$ when divided by the $(x - 1)and$ $(x + 2)$ respectively. Calculate the values of l and m.

Solution.

Let $p(x) = lx^3 + mx^2 - 4$.

By remainder theorem, $p(x)$ is divided by $(x - 1)$, then remainder is 0.

$$p(1) = -3$$
$$l(1)^3 + m(1)^2 - 4 = -3$$
$$l + m = -3 + 4$$
$$l + m = 1$$
$$l = 1 - m - - - (i)$$

By remainder theorem, $p(x)$ is divided by $(x + 2)$, then remainder is 12.

$$p(-2) = 12$$
$$l(-2)^3 + m(-2)^2 - 4 = 12$$
$$-8l + 4m - 4 = 12$$
$$-8l + 4m = 12 + 4$$
$$-8l + 4m = 16$$
$$4(-2l + m) = 16$$
$$-2l + m = \frac{16}{4}$$
$$-2l + m = 4 - - - (ii)$$

Using (i) in (ii) , we have

$$-2(1 - m) + m = 4$$
$$-2 + 2m + m = 4$$
$$3m = 4 + 2$$
$$3m = 6$$
$$m = \frac{6}{3}$$
$$m = 2.$$

Using value of m in (ii), we have

$$-2l + 2 = 4$$
$$-2l = 4 - 2$$
$$-2l = 2$$
$$l = -\frac{2}{2}$$
$$l = -1.$$

Hence $l = -1$ and $m = 2$.

Question.9.

The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible by $x^2 - 5x + 6$. Find the values of a and b.

Solution.

Let $p(x) = ax^3 - 9x^2 + bx + 3a$

As

$$x^2 - 5x + 6 = 0$$
$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$(x-3)(x-2) = 0$$
$$x - 3 = 0, x - 2 = 0$$

By remainder theorem, $p(x)$ is divided by $(x-3)$, then remainder is 0.

$$p(3) = 0$$
$$a(3)^3 - 9(3)^2 + b(3) + 3a = 0$$
$$27a - 81 + 3b + 3a = 0$$
$$30a + 3b = 81 --- (i)$$

By remainder theorem, $p(x)$ is divided by $(x-2)$, then remainder is 0.

$$p(2) = 0$$
$$a(2)^3 - 9(2)^2 + b(2) + 3a = 0$$
$$8a - 36 + 2b + 3a = 0$$
$$11a + 2b = 36 --- (ii)$$

Multiply equation (i) by 2 and equation (ii) by 3, then subtracting eq. (ii) from (i), we have

$$60a + 6b = 162$$
$$\pm 33a \pm 6b = \pm 108$$

$$27a = 54$$
$$a = 2$$

Put $a = 2$ *in eq.* (i), *we get*

$$30(2) + 3b = 81$$
$$60 + 3b = 81$$
$$3b = 81 - 60$$
$$3b = 21$$
$$b = 7$$

Hence $a = 2$ *and* $b = 7$.

Rational Root Theorem

Let

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n, \quad a_n \neq 0 \ldots\ldots$ (i)

Be a polynomial equation of degree n with integral coefficients. If $\frac{p}{q}$ is a rational root of the equation, then p is a factor of the constant term a_n and q is the factor of *leading coefficient* a_0.

EXERCISE# 5.4

Factorize each of the following cubic polynomials by factor theorem.

Q#1) $\boldsymbol{x^3 - 2x^2 - x + 2}$

Solution: Let $P(x) = x^3 - 2x^2 - x + 2 \ldots (1)$

Here, the constant term is 2 and factors of constant terms are $\pm1, \pm2, \pm4, \ldots$

Therefore, we check $\pm1, \pm2, \pm4$ for the roots.

Now, put $x = 1$ in (1), we have

$P(1) = (1)^3 - 2(1)^2 - (1) + 2$

$P(1) = 1 - 2 - 1 + 2$

$P(1) = 0$

Hence, $x = 1$ is the root of $P(x)$, therefore $(x-1)$ is the factor of $P(x)$.

Now, put $x = 2$ in (1), we have

$P(2) = (2)^3 - 2(2)^2 - (2) + 2$

$P(2) = 8 - 8 - 2 + 2$

$P(2) = 0$

Hence, $x = 2$ is the root of $P(x)$, therefore $(x-2)$ is the factor of $P(x)$.

Now, put $x = -1$ in (1), we have

$P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$P(-1) = -1 - 2 + 1 + 2$

$P(-1) = 0$

Hence, $x = -1$ is the root of $P(x)$, therefore $\big(x - (-1)\big) = (x+1)$ is the factor of $P(x)$.

Thus, $P(x) = (x-1)(x-2)(x+1)$

Q#2) $\boldsymbol{x^3 - x^2 - 22x + 40}$

Solution: Let $P(x) = x^3 - x^2 - 22x + 40 \ldots (1)$

Here, the constant term is 40 and factors of constant terms are $\pm1, \pm2, \pm4, \pm5 \ldots$

Therefore, we check $\pm1, \pm2, \pm4, \pm5$ for the roots.

Now, put $x = 1$ in (1), we have

$P(1) = (1)^3 - (1)^2 - 22(1) + 40$

$P(1) = 1 - 1 - 22 + 40$

$P(1) = 18 \neq 0$

Hence, $x = 1$ is the not the root of $P(x)$,

Now, put $x = 2$ in (1), we have

$P(2) = (2)^3 - (2)^2 - 22(2) + 40$

$P(2) = 8 - 4 - 44 + 40$

$$P(2) = 48 - 48 = 0$$

Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = 4$ in (1), we have
$P(4) = (4)^3 - (4)^2 - 22(4) + 40$
$P(4) = 64 - 16 - 88 + 40$
$P(4) = 104 - 104 = 0$
Hence, $x = 4$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = -5$ in (1), we have
$P(-5) = (-5)^3 - (-5)^2 - 22(-5) + 40$
$P(-5) = -125 - 25 + 110 + 40$
$P(-5) = 150 - 150 = 0$
Hence, $x = -5$ is the root of $P(x)$, therefore $\left(x - (-5)\right) = (x + 5)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 2)(x - 4)(x + 5)$

Q#3) $x^3 - 6x^2 + 3x + 10$

Solution: Let $P(x) = x^3 - 6x^2 + 3x + 10 \ldots (1)$
Here, the constant term is 10 and factors of constant terms are $\pm1, \pm2, \pm4, \pm5$ ...
Therefore, we check $\pm1, \pm2, \pm4, \pm5$ for the roots.
Now, put $x = -1$ in (1), we have
$P(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$
$P(-1) = -1 - 6 - 3 + 10$
$P(-1) = 0$
Hence, $x = -1$ is the root of $P(x)$, therefore $(x — 1) = (x + 1)$ is the factor of $P(x)$.
Now, put $x = 2$ in (1), we have
$P(2) = (2)^3 - 6(2)^2 + 3(2) + 10$
$P(2) = 8 - 24 + 6 + 10$
$P(2) = 24 - 24 = 0$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = 5$ in (1), we have
$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10$
$P(5) = 125 - 6(25) + 15 + 10$
$P(5) = 125 - 150 + 25 = 0$
Hence, $x = 5$ is the root of $P(x)$, therefore $(x - 5)$ is the factor of $P(x)$.
Thus, $P(x) = (x + 1)(x - 2)(x - 5)$

Q#4) $x^3 + x^2 - 10x + 8$

Solution: Let $P(x) = x^3 + x^2 - 10x + 8 \ldots (1)$
Here, the constant term is 8 and factors of constant terms are $\pm1, \pm2, \pm4, \ldots$
Therefore, we check $\pm1, \pm2, \pm4$ for the roots.
Now, put $x = 1$ in (1), we have
$P(1) = (1)^3 + (1)^2 - 10(1) + 8$
$P(1) = 1 + 1 - 10 + 8$
$P(1) = 10 - 10 = 0$
Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = 2$ in (1), we have
$P(2) = (2)^3 + (2)^2 - 10(2) + 8$
$P(2) = 8 + 4 - 20 + 8$
$P(2) = 20 - 20$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is also a factor of $P(x)$.
Now, put $x = -4$ in (1), we have
$P(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8$
$P(-4) = -64 + 16 + 40 + 8$
$P(-4) = -64 + 64 = 0$
Hence, $x = -4$ is the root of $P(x)$, therefore $\left(x - (-4)\right) = (x + 4)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 1)(x - 2)(x + 4)$

Q#5) $x^3 - 2x^2 - 5x + 6$

Solution: Let $P(x) = x^3 - 2x^2 - 5x + 6 \ldots (1)$
Here, the constant term is 2 and factors of constant terms are $\pm1, \pm2, \pm3, \ldots$
Therefore, we check $\pm1, \pm2, \pm3$ for the roots.
Now, put $x = 1$ in (1), we have
$P(1) = (1)^3 - 2(1)^2 - 5(1) + 6$
$P(1) = 1 - 2 - 5 + 6$
$P(1) = 7 - 7 = 0$
Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = 3$ in (1), we have
$P(3) = (3)^3 - 2(3)^2 - 5(3) + 6$
$P(2) = 27 - 2(9) - 15 + 6$
$P(2) = 27 - 18 - 15 + 6 = 33 - 33 = 0$
Hence, $x = 3$ is the root of $P(x)$, therefore $(x - 3)$ is the factor of $P(x)$.
Now, put $x = -2$ in (1), we have
$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$
$P(-2) = -8 - 2(4) + 10 + 6$
$P(-2) = -8 - 8 + 10 + 6 = -16 + 16 = 0$
Hence, $x = -2$ is the root of $P(x)$, therefore $\left(x - (-2)\right) = (x + 2)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 1)(x - 3)(x + 2)$

Q#6) $x^3 + 5x^2 - 2x - 24$

Solution: Let $P(x) = x^3 + 5x^2 - 2x - 24 \ldots (1)$
Here, the constant term is 24 and factors of constant terms are $\pm1, \pm2, \pm4, \ldots$
Therefore, we check $\pm1, \pm2, \pm4$ for the roots.
Now, put $x = 2$ in (1), we have
$P(2) = (2)^3 + 5(2)^2 - 2(2) - 24$
$P(2) = 8 + 5(4) - 4 - 24$
$P(2) = 8 + 20 - 4 - 24 = 28 - 28 = 0$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = -3$ in (1), we have
$P(-3) = (-3)^3 + 5(-3)^2 - 2(-3) - 24$
$P(-3) = -27 + 5(9) + 6 - 24$

$P(-3) = -27 + 45 + 6 - 24 = 51 - 51 = 0$
Hence, $x = -3$ is the root of $P(x)$, therefore $(x + 3)$ is the factor of $P(x)$.
Now, put $x = -4$ in (1), we have
$P(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24$
$P(-4) = -64 + 5(16) + 8 - 24$
$P(-4) = -64 + 80 + 8 - 24 = 88 - 88 = 0$
Hence, $x = -4$ is the root of $P(x)$, therefore $\big(x - (-4)\big) = (x + 4)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 2)(x + 3)(x + 4)$

Q#7) $\mathbf{3x^3 - x^2 - 12x + 4}$

Solution: Let $P(x) = 3x^3 - x^2 - 12x + 4 \ldots(1)$
Here, the constant term is 4 and factors of constant terms are $\pm1, \pm2, \pm4, \ldots$
Therefore, we check $\pm1, \pm2, \pm4$ for the roots.
Now, put $x = 2$ in (1), we have
$P(2) = 3(2)^3 - (2)^2 - 12(2) + 4$
$P(2) = 3(8) - 4 - 24 + 4$
$P(2) = 24 - 4 - 24 + 4 = 0$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = -2$ in (1), we have
$P(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4$
$P(-2) = 3(-8) - 4 + 24 + 4$
$P(-2) = -24 - 4 + 24 + 4 = 0$
Hence, $x = -2$ is the root of $P(x)$, therefore $\big(x - (-2)\big) = (x + 2)$ is the factor of $P(x)$.
Since the leading co-efficient is 3, therefore we also check at $x = \frac{1}{3}$ and $x = -\frac{1}{3}$
First we check at $x = \frac{1}{3}$

$$P\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right) + 4$$
$$P\left(\frac{1}{3}\right) = 3\left(\frac{1}{27}\right) - \left(\frac{1}{9}\right) - 4 + 4$$
$$P\left(\frac{1}{3}\right) = \frac{1}{9} - \frac{1}{9} - 4 + 4 = 0$$

Hence, $x = \frac{1}{3}$ is the root of $P(x)$, therefore $\left(x - \frac{1}{3}\right) = 0$ gives that $(3x - 1)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 2)(x + 2)(3x - 1)$

Q#8) $\mathbf{2x^3 + x^2 - 2x - 1}$

Solution: Let $P(x) = 2x^3 + x^2 - 2x - 1 \ldots(1)$
Here, the constant term is 1 and factors of constant terms are ±1 .
Therefore, we check ±1 for the roots.
Now, put $x = 1$ in (1), we have
$P(1) = 2(1)^3 + (1)^2 - 2(1) - 1$
$P(1) = 2 + 1 - 2 - 1$
$P(1) = 3 - 3 = 0$
Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = -1$ in (1), we have
$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$P(-1) = -2 + 1 + 2 - 1$
$P(-1) = -3 + 3 = 0$
Hence, $x = -1$ is the root of $P(x)$, therefore $\big(x - (-1)\big) = (x + 1)$ is the factor of $P(x)$.
Since the leading co-efficient is 2, therefore we check $x = -\frac{1}{2}$ and $x = \frac{1}{2}$
First we check at $x = -\frac{1}{2}$

$$P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 1$$
$$P\left(-\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) - \left(\frac{1}{4}\right) + 1 - 1$$
$$P\left(-\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{4} + 1 - 1 = 0$$

Hence, $x = -\frac{1}{2}$ is the root of $P(x)$, therefore $\left(x + \frac{1}{2}\right) = 0$ gives that $(2x + 1)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 1)(x + 1)(2x + 1)$

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Chapter 7.

LINEAR EQUATIONS AND INEQUALITIES

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Contents

Radical Equation

When the variable in an equation occurs under a radical, the equation is called a radical equation. For example,

$$\sqrt{x-3}-7=0$$

Linear Equation

A linear equation in one unknown variable x is an equation of the form $ax + b = 0$, where $a, b \in R \text{ and } a \neq 0$

A solution to a linear equation is any replacement or substitution for the variable x that makes the statement true. Two linear equations are said to be equivalent if they have exactly the same solution. For example, $x+1=0,\ \ 2x+5=-1$

EXERCISE 7.1

Q#1) Solve the following equations.

(i). $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Solution: As given $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Multiply by 6(LCM) on both sides

$$6\times\frac{2}{3}x-6\times\frac{1}{2}x=6\times x+6\times\frac{1}{6}$$
$$4x-3x=6x+1$$
$$x=6x+1$$
$$1=x-6x$$
$$1=-5x$$
$$x=-\frac{1}{5}$$

Check:

$$\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$$

Put $x=-\frac{1}{5}$

$$\frac{2}{3}\left(-\frac{1}{5}\right)-\frac{1}{2}\left(-\frac{1}{5}\right)=\left(-\frac{1}{5}\right)+\frac{1}{6}$$
$$-\frac{2}{15}+\frac{1}{10}=-\frac{1}{5}+\frac{1}{6}$$

Multiply by 30(LCM) on both sides

$$30\times\left(-\frac{2}{15}\right)+30\times\left(\frac{1}{10}\right)$$
$$=30\times\left(-\frac{1}{5}\right)+30\times\left(\frac{1}{6}\right)$$
$$-4+3=-6+5$$
$$-1=-1(\text{which is true})$$

Since $x=-\frac{1}{5}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{5}\right\}$ i.e. $S.S=\left\{-\frac{1}{5}\right\}$

(ii). $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Solution: As given $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Multiply by 6(LCM) on both sides

$$6\times\left(\frac{x-3}{3}\right)-6\times\left(\frac{x-2}{2}\right)=6\times(-1)$$
$$2(x-3)-3(x-2)=-6$$
$$2x-6-3x+6=-6$$
$$-x=-6$$
$$x=6$$

Check:

$$\frac{x-3}{3}-\frac{x-2}{2}=-1$$

Put $x=6$

$$\frac{(6)-3}{3}-\frac{(6)-2}{2}=-1$$
$$\frac{3}{3}-\frac{4}{2}=-1$$
$$1-2=-1$$
$$-1=-1\ (\text{which is true})$$

Since $x=6$ satisfy the given equation, therefore, the solution set is $\{6\}$ i.e. $S.S=\{6\}$

(ii). $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Solution:

As given $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

$$\frac{1}{2}x-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-x$$

Multiply by 12(LCM) on both sides

$$12\times\left(\frac{1}{2}x\right)-12\times\left(\frac{1}{12}\right)$$
$$+12\times\left(\frac{2}{3}\right)=12\times\left(\frac{5}{6}\right)+12\times\left(\frac{1}{6}\right)-12\times(x)$$
$$6x-1+8=10+2-12x$$
$$6x+7=12-12x$$
$$6x+12x=12-7$$
$$18x=5$$
$$x=\frac{5}{18}$$

Check:

$$\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$$
$$\frac{1}{2}x-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-x$$

Put $x=\frac{5}{18}$

$$\frac{1}{2}\left(\frac{5}{18}\right)-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-\left(\frac{5}{18}\right)$$
$$\frac{5}{36}-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-\frac{5}{18}$$

Multiply by 36(LCM) on both sides

$$36\times\left(\frac{5}{36}\right)-36\times\left(\frac{1}{12}\right)$$
$$+36\times\left(\frac{2}{3}\right)=36\times\left(\frac{5}{6}\right)+36\times\left(\frac{1}{6}\right)-36\times\left(\frac{5}{18}\right)$$

$$5-3+24=30+6-10$$
$$-3+29=36-10$$
$$26=26 \quad \text{(which is true)}$$

Since $x=\frac{5}{18}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{5}{18}\right\}$ i.e. $S.S=\left\{\frac{5}{18}\right\}$

(iv). $\boldsymbol{x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x}$

Solution: As given $x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$

$$x+\frac{1}{3}=2x-\frac{4}{3}-6x$$

Multiply by 3(LCM) on both sides

$$3\times(x)+3\times\left(\frac{1}{3}\right)$$
$$=3\times(2x)-3\times\left(\frac{4}{3}\right)-3\times(6x)$$
$$3x+1=6x-4-18x$$
$$3x+1=-4-12x$$
$$3x+12=-4-1$$
$$15x=-5$$
$$x=-\frac{5}{15}$$
$$x=-\frac{1}{3}$$

Check:

$$x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$$
$$x+\frac{1}{3}=2x-\frac{4}{3}-6x$$

Put $x=-\frac{1}{3}$

$$\left(-\frac{1}{3}\right)+\frac{1}{3}=2\left(-\frac{1}{3}\right)-\frac{4}{3}-6\left(-\frac{1}{3}\right)$$
$$-\frac{1}{3}+\frac{1}{3}=-\frac{2}{3}-\frac{4}{3}+2$$

Multiply by 3(LCM) on both sides

$$-1+1=-2-4+6$$
$$0=0 \text{ (which is true)}$$

Since $x=-\frac{1}{3}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{3}\right\}$ i.e. $S.S=\left\{-\frac{1}{3}\right\}$

(v) $\boldsymbol{\frac{5(x-3)}{6}-x=1-\frac{x}{9}}$

Solution: As given $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Multiply by 18 (LCM) on both sides

$$18\times\left(\frac{5(x-3)}{6}\right)-18\times(x)$$
$$=18\times(1)-18\times\left(\frac{x}{9}\right)$$
$$3(5x-15)-18x=18-2x$$
$$15x-45-18x=18-2x$$
$$-45-3x=18-2x$$
$$-3x+2x=18+45$$
$$-x=63$$
$$x=-63$$

Check:

$$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$$

Put $x=-63$

$$\frac{5(-63-3)}{6}-(-63)=1-\frac{(-63)}{9}$$
$$\frac{5(-66)}{6}+63=1+\frac{63}{9}$$
$$-55+63=1+7$$
$$8=8 \text{ (which is true)}$$

Since $x=-63$ satisfy the given equation, therefore, the solution set is $\{-63\}$ i.e. $S.S=\{-63\}$

(vi). $\boldsymbol{\frac{x}{3x-6}=2-\frac{2x}{x-2},\ x\neq 2}$

Solution: As given $\frac{x}{3x-6}=2-\frac{2x}{x-2}$

$$\frac{x}{3x-6}=\frac{2(x-2)-2x}{x-2}$$
$$\frac{x}{3x-6}=\frac{2x-4-2x}{x-2}$$
$$\frac{x}{3x-6}=\frac{-4}{x-2}$$
$$x(x-2)=-4(3x-6)$$
$$x^2-2x=-12x+24$$
$$x^2-2x+12x-24=0$$
$$x(x-2)+12(x-2)=0$$
$$(x-2)(x+12)=0$$

That is $x=2,-12$

Since it is given that $x\neq 2$, therefore, we ignore $x=2$ and just check $x=-12$ for the solution set.

Check:

$$\frac{x}{3x-6}=2-\frac{2x}{x-2}$$

Put $x=-12$

$$\frac{(-12)}{3(-12)-6}=2-\frac{2(-12)}{(-12)-2}$$
$$\frac{-12}{-36-6}=2+\frac{24}{-12-2}$$
$$\frac{-12}{-42}=2+\frac{24}{-14}$$
$$\frac{2}{7}=\frac{28-24}{14}$$
$$\frac{2}{7}=\frac{4}{14}$$
$$\frac{2}{7}=\frac{2}{7} \quad \text{(which is true)}$$

Since $x=-12$ satisfy the given equation, therefore, the solution set is $\{-12\}$ i.e. $S.S=\{-12\}$

(vii). $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}$, $x \neq \frac{6}{2}$

Solution: As given $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}$

$$\frac{2x}{2x+5} = \frac{2(4x+10)-15}{3(4x+10)}$$
$$\frac{2x}{2x+5} = \frac{8x+20-15}{12x+30}$$
$$\frac{2x}{2x+5} = \frac{8x+5}{12x+30}$$
$$2x(12x+30) = (8x+5)(2x+5)$$
$$24x^2+60x = 16x^2+40x+10x+25$$
$$24x^2+60x = 16x^2+50x+25$$
$$24x^2+60x-16x^2-50x-25 = 0$$
$$8x^2+10x-25 = 0$$
$$8x^2+20x-10x-25 = 0$$
$$4x(2x+5)-5(2x+5) = 0$$
$$(2x+5)(4x-5) = 0$$

That is $x = -\frac{5}{2}, \frac{5}{4}$

Since it is given that $x \neq -\frac{5}{2}$, therefore, we ignore $x = -\frac{5}{2}$ and just check $x = \frac{5}{4}$ for the solution set.

Check:

$$\frac{2x}{2x+5} = \frac{2}{3} - \frac{5x}{4x+10}$$

Put $x = \frac{5}{4}$

$$\frac{2\left(\frac{5}{4}\right)}{2\left(\frac{5}{4}\right)+5} = \frac{2}{3} - \frac{5}{4\left(\frac{5}{4}\right)+10}$$
$$\frac{\frac{5}{2}}{\frac{5}{2}+5} = \frac{2}{3} - \frac{5}{5+10}$$
$$\frac{\frac{5}{2}}{\frac{5+10}{2}} = \frac{2}{3} - \frac{5}{15}$$
$$\frac{5}{15} = \frac{2}{3} - \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3} \quad \text{(which is true)}$$

Since $x = \frac{5}{4}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{5}{4}\right\}$ i.e. $S.S = \left\{\frac{5}{4}\right\}$

(viii). $\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}$, $x \neq 1$

Solution: As given $\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}$

$$\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}$$
$$\frac{3(2x)+(x-1)}{3(x-1)} = \frac{5(x-1)+2(6)}{6(x-1)}$$
$$\frac{6x+x-1}{3(x-1)} = \frac{5x-5+12}{6(x-1)}$$
$$\frac{7x-1}{3(x-1)} = \frac{5x+7}{6(x-1)}$$
$$(7x-1)6(x-1) = 3(x-1)(5x+7)$$
$$6(7x-1)(x-1) - 3(x-1)(5x+7) = 0$$
$$3(x-1)[2(7x-1)-(5x+7)] = 0$$
$$3(x-1)(14x-2-5x-7) = 0$$
$$3(x-1)(9x-9) = 0$$
$$3(x-1)9(x-1) = 0$$
$$27(x-1)^2 = 0$$

Which implies that

$(x-1) = 0$ gives that $x = 1$ which is not possible (given $x \neq 1$)

therefore, the solution set is {} i.e. $S.S = \{\}$

(ix). $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}$, $x \neq \pm 1$

Solution: As given $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}$

$$\frac{2}{(x+1)(x-1)} - \frac{1}{x+1} = \frac{1}{x+1}$$
$$\frac{2-(x-1)}{(x+1)(x-1)} = \frac{1}{x+1}$$
$$\frac{2-x+1}{(x+1)(x-1)} = \frac{1}{x+1}$$
$$\frac{3-x}{(x+1)(x-1)} = \frac{1}{x+1}$$
$$(3-x)(x+1) = (x+1)(x-1)$$
$$(3-x)(x+1) - (x+1)(x-1) = 0$$
$$(x+1)[(3-x)-(x-1)] = 0$$
$$(x+1)(3-x-x+1) = 0$$
$$(x+1)(4-2x) = 0$$

That is $x = -1, 2$

Since it is given that $x \neq \pm 1$, therefore, we ignore $x = -1$ and just check $x = 2$ for the solution set.

Check:

$$\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}$$

Put $x = 2$

$$\frac{2}{(2)^2-1} - \frac{1}{(2)+1} = \frac{1}{(2)+1}$$
$$\frac{2}{4-1} - \frac{1}{3} = \frac{1}{3}$$
$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3} \quad \text{(which is true)}$$

Since $x = 2$ satisfy the given equation, therefore, the solution set is {2} i.e. $S.S = \{2\}$

(x). $\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$, $x \neq -2$

Solution: As given $\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$

$$\frac{2}{3x+6} = \frac{1(2x+4)-6}{6(2x+4)}$$

$$\frac{2}{3x+6}=\frac{2x+4-6}{6(2x+4)}$$
$$\frac{2}{3x+6}=\frac{2x-2}{6(2x+4)}$$
$$\frac{2}{3(x+2)}=\frac{2(x-1)}{6(2(x+2))}$$
$$\frac{2}{3(x+2)}=\frac{(x-1)}{6(x+2)}$$
$$12(x+2)=3(x+2)(x-1)$$
$$12(x+2)-3(x+2)(x-1)=0$$

$$3(x+2)[4-x+1)]=0$$
$$3(x+2)(5-x)=0$$

That is $x=-2,5$

Since it is given that $x\neq-2$, therefore, we ignore $x=-2$ and just check $x=5$ for the solution set.

Check:

$$\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$$

Put $x=5$

$$\frac{2}{3(5)+6}=\frac{1}{6}-\frac{1}{2(5)+4}$$
$$\frac{2}{15+6}=\frac{1}{6}-\frac{1}{10+4}$$
$$\frac{2}{21}=\frac{1}{6}-\frac{1}{14}$$
$$\frac{2}{21}=\frac{7-3}{42}$$
$$\frac{2}{21}=\frac{2}{42}$$

$\frac{2}{21}=\frac{2}{21}$ (which is true)

Since $x=5$ satisfy the given equation, therefore, the solution set is $\{5\}$ i.e. $S.S=\{5\}$

Q#2) Solve each equation and check for extraneous solution if any.

(i). $\sqrt{3x+4}=2$

Solution: As given $\sqrt{3x+4}=2$

On squaring, we get

$$\left(\sqrt{3x+4}\right)^2=(2)^2$$
$$3x+4=4$$
$$3x=0$$
$$x=0$$

Check:

$$\sqrt{3x+4}=2$$

Put $x=0$

$$\sqrt{3(0)+4}=2$$
$$\sqrt{0+4}=2$$

$2=2$ (which is true)

Since $x=0$ satisfy the given equation, therefore, the solution set is $\{0\}$ i.e. $S.S=\{0\}$

(ii). $\sqrt[3]{2x-4}-2=0$

Solution: As given $\sqrt[3]{2x-4}-2=0$

$$\sqrt[3]{2x-4}=2$$

Taking cube on both sides

$$\left(\sqrt[3]{2x-4}\right)^3=(2)^3$$
$$2x-4=8$$
$$2x=8+4$$
$$2x=12$$
$$x=6$$

Check:

$$\sqrt[3]{2x-4}-2=0$$

Put $x=6$

$$\sqrt[3]{2(6)-4}-2=0$$
$$\sqrt[3]{12-4}-2=0$$
$$\sqrt[3]{8}-2=0$$
$$2-2=0$$

$0=0$ (which is true)

Since $x=6$ satisfy the given equation, therefore, the solution set is $\{6\}$ i.e. $S.S=\{6\}$

(ii). $\sqrt{x-3}-7=0$

Solution: As given $\sqrt{x-3}-7=0$

$$\sqrt{x-3}=7$$

Taking square on both sides

$$\left(\sqrt{x-3}\right)^2=(7)^2$$
$$x-3=49$$
$$x=49+3$$
$$x=52$$

Check:

$$\sqrt{x-3}-7=0$$

Put $x=52$

$$\sqrt{52-3}-7=0$$
$$\sqrt{49}-7=0$$
$$7-7=0$$

$0=0$ (which is true)

Since $x=52$ satisfy the given equation, therefore, the solution set is $\{52\}$ i.e. $S.S=\{52\}$

(iii). $2\sqrt{t+4}=5$

Solution: As given $2\sqrt{t+4}=5$

Taking saquare on both sides

$$\left(2\sqrt{t+4}\right)^2=(5)^2$$
$$4(t+4)=25$$
$$4t+16=25$$
$$4t=25-16$$
$$t=\frac{9}{4}$$

Check:

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$$2\sqrt{t+4} = 5$$

Put $t = \frac{9}{2}$

$$2\sqrt{\frac{9}{4}+4} = 5$$

$$2\sqrt{\frac{9+16}{4}} = 5$$

$$2\sqrt{\frac{25}{4}} = 5$$

$$2\left(\frac{5}{2}\right) = 5$$

$$5 = 5 \quad \text{(which is true)}$$

Since $t = \frac{9}{4}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{9}{4}\right\}$ i.e. $S.S = \left\{\frac{9}{4}\right\}$

(v). $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Solution: As given $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Taking cube on both sides

$$\left(\sqrt[3]{2x+3}\right)^3 = \left(\sqrt[3]{x-2}\right)^3$$

$$2x+3 = x-2$$

$$2x - x = -2-3$$

$$x = -5$$

Check:

$$\sqrt[3]{2x+3} = \sqrt[3]{x-2}$$

Put $x = -5$

$$\sqrt[3]{2(-5)+3} = \sqrt[3]{(-5)-2}$$

$$\sqrt[3]{-10+3} = \sqrt[3]{-5-2}$$

$$\sqrt[3]{-7} = \sqrt[3]{-7}$$

Taking cube root, we have

$$-7 = -7 \text{ (which is true)}$$

Since $x = -5$ satisfy the given equation, therefore, the solution set is $\{-5\}$ i.e. $S.S = \{-5\}$

(v). $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Solution: As given $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Taking cube on both sides

$$\left(\sqrt[3]{2-t}\right)^3 = \left(\sqrt[3]{2t-28}\right)^3$$

$$2-t = 2t-28$$

$$2+28 = 2t+t$$

$$30 = 3t$$

$$t = 10$$

Check:

$$\sqrt[3]{2-t} = \sqrt[3]{2t-28}$$

Put $t = 10$

$$\sqrt[3]{2-10} = \sqrt[3]{2(10)-28}$$

$$\sqrt[3]{-8} = \sqrt[3]{20-28}$$

$$\sqrt[3]{-8} = \sqrt[3]{-8}$$

Taking cube root, we have

$$-8 = -8 \text{ (which is true)}$$

Since t= 10 satisfy the given equation, therefore, the solution set is $\{10\}$ i.e. $S.S = \{10\}$

(viii). $\sqrt{\frac{x+1}{2x+5}} = 2,\ x \neq -\frac{5}{2}$

Solution: As given $\sqrt{\frac{x+1}{2x+5}} = 2$

Taking square on both sides

$$\left(\sqrt{\frac{x+1}{2x+5}}\right)^2 = (2)^2$$

$$\frac{x+1}{2x+5} = 4$$

$$x+1 = 4(2x+5)$$

$$x+1 = 8x+20$$

$$1-20 = 8x-x$$

$$-19 = 7x$$

$$x = -\frac{19}{7}$$

Check:

$$\sqrt{\frac{x+1}{2x+5}} = 2$$

Put $x = -\frac{19}{7}$

$$\sqrt{\frac{\left(-\frac{19}{7}\right)+1}{2\left(-\frac{19}{7}\right)+5}} = 2$$

$$\sqrt{\frac{\frac{-19+7}{7}}{\frac{-38+35}{7}}} = 2$$

$$\sqrt{\frac{\frac{-12}{7}}{\frac{-3}{7}}} = 2$$

$$\sqrt{\frac{12}{3}} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2 \text{ (which is true)}$$

Since $x = -\frac{19}{7}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{19}{7}\right\}$ i.e. $S.S = \left\{-\frac{19}{7}\right\}$

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Absolute Value

The Absolute value of real number $'a'$ is denoted by $|a|$, is defined as

$$|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$$

For example, $|6| = 6$, $|-5| = -(-5) = 5$

$$|0| = 0$$

Some Properties of Absolute value

If $a, b \in R$, then

(i). $|a| \geq 0$

(ii). $|-a| = |a|$

(iii). $|ab| = |a||b|$

(iv). $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $|b| \neq 0$

EXERCISE 7.2

Q#1) **1. Identify the following statements as True or False.**

(i) $|x| = 0$ has only one solution. ⋯T⋯

(ii) All absolute value equations have two solutions. F ⋯

(iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$. ⋯T⋯⋯

(iv) The equation $|x - 4| = -4$ has no solution. ⋯F

(v) The equation $|2x - 3| = 5$ is equivalent to $2x - 3 = 5$ or $2x + 3 = 5$. ⋯F ⋯

Q#2) Solve for x

(i). $|3x - 5| = 4$

Sol: As given $|3x - 5| = 4$

By definition, we have

$3x - 5 = 4$ or $3x - 5 = -4$

$3x = 4 + 5$ or $3x = -4 + 5$

$3x = 9$ or $3x = 1$

$x = 3$ or $x = \frac{1}{3}$

Check:

$$|3x - 5| = 4 \ldots(1)$$

Put $x = 3$, in (1)

$$|3(3) - 5| = 4$$
$$|9 - 5| = 4$$
$$|4| = 4$$
$$4 = 4 \text{ (which is true)}$$

Put $x = \frac{1}{3}$, in (1)

$$\left|3\left(\frac{1}{3}\right) - 5\right| = 4$$
$$|1 - 5| = 4$$
$$|-4| = 4$$
$$4 = 4 \text{ (which is true)}$$

Since $x = 3, \frac{1}{3}$ satisfy the given equation, therefore, the solution set is $\left\{3, \frac{1}{3}\right\}$ i.e. $S.S = \left\{3, \frac{1}{3}\right\}$

(ii). $\frac{1}{2}|3x + 2| - 4 = 11$

Solution: As given $\frac{1}{2}|3x + 2| - 4 = 11$

$$\frac{1}{2}|3x + 2| = 11 + 4$$
$$\frac{1}{2}|3x + 2| = 15$$
$$|3x + 2| = 30$$

By definition, we have

$3x + 2 = 30$ or $3x + 2 = -30$

$3x = 30 - 2$ or $3x = -30 - 2$

$3x = 28$ or $3x = -\frac{32}{3}$

$x = \frac{28}{3}$ or $x = -\frac{32}{3}$

Check:

$$\frac{1}{2}|3x + 2| - 4 = 11 \ldots(1)$$

Put $x = \frac{28}{3}$, in (1)

$$\frac{1}{2}\left|3\left(\frac{28}{3}\right) + 2\right| - 4 = 11$$
$$\frac{1}{2}|28 + 2| - 4 = 11$$
$$\frac{1}{2}|30| - 4 = 11$$
$$\frac{1}{2}(30) - 4 = 11$$
$$15 - 4 = 11$$
$$11 = 11 \text{ (which is true)}$$

Put $x = -\frac{32}{3}$, in (1)

$$\frac{1}{2}\left|3\left(-\frac{32}{3}\right) + 2\right| - 4 = 11$$
$$\frac{1}{2}|-32 + 2| - 4 = 11$$
$$\frac{1}{2}|-30| - 4 = 11$$
$$\frac{1}{2}(30) - 4 = 11$$
$$15 - 4 = 11$$
$$11 = 11 \text{ (which is true)}$$

Since $x = \frac{28}{3}, -\frac{32}{3}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{28}{3}, -\frac{32}{3}\right\}$ i.e. $S.S = \left\{\frac{28}{3}, -\frac{32}{3}\right\}$

(iii). $|2x + 5| = 11$

Solution: As given $|2x + 5| = 11$

By definition, we have

$2x + 5 = 11$ or $2x + 5 = -11$

$2x = 11 - 5$ or $2x = -11 - 5$

$2x = 6$ or $2x = -16$

$x = 3$ or $x = -8$

Check:

$$|2x + 5| = 11 \dots (1)$$

Put $x = 3$, in (1)

$$|2(3) + 5| = 11$$
$$|6 + 5| = 11$$
$$|11| = 11$$
$$11 = 11 \text{ (which is true)}$$

Put $x = -8$, in (1)

$$|2(-8) + 5| = 11$$
$$|-16 + 5| = 11$$
$$|-11| = 11$$
$$11 = 11 \text{ (which is true)}$$

Since $x = 3, -8$ satisfy the given equation, therefore, the solution set is $\{3, -8\}$ i.e. $S.S = \{3, -8\}$

(iii). $|3 + 2x| = |6x - 7|$

Solution: As given $|3 + 2x| = |6x - 7|$

$$\frac{|3 + 2x|}{|6x - 7|} = 1$$
$$\left|\frac{3 + 2x}{6x - 7}\right| = 1$$

By definition, we have

$\frac{3+2x}{6x-7} = 1$ or $\frac{3+2x}{6x-7} = -1$

$3 + 2x = 6x - 7$ or $3 + 2x = -6x + 7$

$3 + 7 = 6x - 2x$ or $2x + 6x = 7 - 3$

$10 = 4x$ or $8x = 4$

$x = \frac{5}{2}$ or $x = \frac{1}{2}$

Check:

$$|3 + 2x| = |6x - 7| \dots (1)$$

Put $x = \frac{5}{2}$, in (1)

$$\left|3 + 2\left(\frac{5}{2}\right)\right| = \left|6\left(\frac{5}{2}\right) - 7\right|$$
$$|3 + 5| = |15 - 7|$$
$$|8| = |8|$$
$$8 = 8 \text{ (which is true)}$$

Put $x = \frac{1}{2}$, in (1)

$$\left|3 + 2\left(\frac{1}{2}\right)\right| = \left|6\left(\frac{1}{2}\right) - 7\right|$$
$$|3 + 1| = |3 - 7|$$
$$|4| = |-4|$$
$$4 = 4 \text{ (which is true)}$$

Since $x = \frac{5}{2}, \frac{1}{2}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{5}{2}, \frac{1}{2}\right\}$ i.e. $S.S = \left\{\frac{5}{2}, \frac{1}{2}\right\}$

(v). $|x + 2| - 3 = 5 - |x + 2|$

Solution: As given $|x + 2| - 3 = 5 - |x + 2|$

$$|x + 2| + |x + 2| = 5 + 3$$
$$2|x + 2| = 8$$
$$|x + 2| = 4$$

By definition, we have

$x + 2 = 4$ or $x + 2 = -4$

$x = 4 - 2$ or $x = -4 - 2$

$x = 2$ or $x = -6$

Check:

$$|x + 2| - 3 = 5 - |x + 2| \dots (1)$$

Put $x = 2$, in (1)

$$|2 + 2| - 3 = 5 - |2 + 2|$$
$$|4| - 3 = 5 - |4|$$
$$4 - 3 = 5 - 4$$
$$1 = 1 \text{ (which is true)}$$

Put $x = -6$, in (1)

$$|-6 + 2| - 3 = 5 - |-6 + 2|$$
$$|-4| - 3 = 5 - |-4|$$
$$4 - 3 = 5 - 4$$
$$1 = 1 \text{ (which is true)}$$

Since $x = 2, -6$ satisfy the given equation, therefore, the solution set is $\{2, -6\}$ i.e. $S.S = \{2, -6\}$

(vi). $\frac{1}{2}|x + 3| + 21 = 9$

Solution: As given $\frac{1}{2}|x + 3| + 21 = 9$

$$\frac{1}{2}|x + 3| = 9 - 21$$
$$\frac{1}{2}|x + 3| = -12$$
$$|x + 3| = -24$$

Which is not possible as modulus value is always non-negative.

(vii). $\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

Sol: As given $\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$$\left|\frac{3 - 5x}{4}\right| = \frac{2}{3} + \frac{1}{3}$$
$$\left|\frac{3 - 5x}{4}\right| = \frac{2 + 1}{3}$$
$$\left|\frac{3 - 5x}{4}\right| = \frac{3}{3}$$
$$\left|\frac{3 - 5x}{4}\right| = 1$$
$$|3 - 5x| = 4$$

By definition, we have

$3 - 5x = 4$ or $3 - 5x = -4$

$3 - 4 = 5x$ or $3 + 4 = 5x$

$-1 = 5x$ or $7 = 5x$

$x = -\frac{1}{5}$ or $x = \frac{7}{5}$

Check:

$$\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3} \dots (1)$$

Put $x = -\frac{1}{5}$, in (1)

$$\left|\frac{3-5\left(-\frac{1}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3+1}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$|1|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3}\text{(which is true)}$$

Put $x=\frac{7}{5}$, in (1)

$$\left|\frac{3-5\left(\frac{7}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3-7}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{-4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$|-1|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3}\text{(which is true)}$$

Since $x=-\frac{1}{5},\frac{7}{5}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{5},\frac{7}{5}\right\}$ i.e. $S.S=\left\{-\frac{1}{5},\frac{7}{5}\right\}$

(viii). $\left|\frac{x+5}{2-x}\right|=6$

Solution: As given $\left|\frac{x+5}{2-x}\right|=6$

By definition, we have

$\frac{x+5}{2-x}=6$ or $\frac{x+5}{2-x}=-6$

$x+5=12-6x$ or $x+5=-12+6x$

$x+6x=12-5$ or $5+12=6x-x$

$7x=7$ or $17=5x$

$x=1$ or $x=\frac{17}{5}$

Check:

$$\left|\frac{x+5}{2-x}\right|=6\ldots(1)$$

Put $x=1$, in (1)

$$\left|\frac{1+5}{2-1}\right|=6$$
$$\left|\frac{6}{1}\right|=6$$
$$|6|=6$$
$$6=6\text{ (which is true)}$$

Put $x=\frac{17}{5}$, in (1)

$$\left|\frac{\left(\frac{17}{5}\right)+5}{2-\left(\frac{17}{5}\right)}\right|=6$$
$$\left|\frac{\frac{17+25}{5}}{\frac{10-17}{5}}\right|=6$$
$$\left|\frac{\frac{42}{5}}{\frac{-7}{5}}\right|=6$$
$$\left|\frac{42}{-7}\right|=6$$
$$|-6|=6$$
$$6=6\text{ (which is true)}$$

Since $x=1,\frac{17}{5}$ satisfy the given equation, therefore, the solution set is $\left\{1,\frac{17}{5}\right\}$ i.e. $S.S=\left\{1,\frac{17}{5}\right\}$

Absolute Value

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form

$$ax + b < 0, a \neq 0$$

where a and b are real numbers. We may replace the symbol $<$ *by* $>, \leq, \geq$ also.

EXERCISE 7.3

Q#1) Solve the following inequalities.

(i). $\mathbf{3x + 1 < 5x - 4}$

Solution: As given $3x + 1 < 5x - 4$

$\Rightarrow\ 5 < 2x$

$\Rightarrow \dfrac{5}{2} < x$

Hence, $S.S = \{x | x > \frac{5}{2}\}$

(ii). $\mathbf{4x - 10.3 \leq 21x - 1.8}$

Solution: As given $4x - 10.3 \leq 21x - 1.8$

$\Rightarrow\ -10.3 + 1.8 \leq 21x - 4x$

$\Rightarrow\ -8.5 \leq 17x$

$\Rightarrow\ -\dfrac{8.5}{15} \leq x$

$\Rightarrow\ x \geq -0.5$

Hence, $S.S = \{x | x \geq -0.5\}$

(iii). $\mathbf{4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x}$

Solution: As given $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

Multiply by 4

$\Rightarrow\ 16 - 2x \geq -28 + x$

$\Rightarrow\ 16 + 28 \geq\ x + 2x$

$\Rightarrow\ 44 \geq 3x$

$\Rightarrow\ x \leq \dfrac{44}{3}$

Hence, $S.S = \{x | x \leq \frac{44}{3}\}$

(iv). $\mathbf{x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}}$

Solution: As given $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

$x - 10 + 4x \geq 6x - \dfrac{7}{2}$

$5x - 10 \geq 6x - \dfrac{7}{2}$

Multiply by 2

$\Rightarrow\ 10x - 20 \geq 12x - 7$

$\Rightarrow\ -20 + 7 \geq\ 12x - 10x$

$\Rightarrow\ -13 \geq 2x$

$\Rightarrow\ x \leq \dfrac{-13}{2}$

$\Rightarrow\ x \leq -6.5$

Hence, $S.S = \{x | x \leq -6.5\}$

(v). $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Sol: As given $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Multiply by 4 (LCM), we have

$\Rightarrow 9 \times \left(\dfrac{3x+2}{9}\right) - 9 \times \left(\dfrac{2x+1}{3}\right) > 9 \times (-1)$

$\Rightarrow (3x + 2) - 3(2x + 1) > -9$

$\Rightarrow\ 3x + 2 - 6x - 3 > -9$

$\Rightarrow\ -3x - 1 > -9$

$\Rightarrow\ -1 + 9 > 3x$

$\Rightarrow\ 8 > 3x$

$\Rightarrow \dfrac{8}{3} > x$

Hence, $S.S = \{x | x < \frac{8}{3}\}$

(vi). $\mathbf{3(2x + 1) - 2(2x + 5) < 5(3x - 2)}$

Solution: As given $3(2x + 1) - 2(2x + 5) < 5(3x - 2)$

$\Rightarrow\ 6x + 3 - 4x - 10 < 15x - 10$

$\Rightarrow\ 2x - 7 < 15x - 10$

$\Rightarrow\ -7 + 10 < 15x - 2x$

$\Rightarrow\ 3 < 13x$

$\Rightarrow \dfrac{3}{13} < x$

Hence, $S.S = \{x | x > \frac{3}{13}\}$

(vii). $\mathbf{3(x - 1) - (x - 2) > -2(x + 4)}$

Solution: As given $3(x - 1) - (x - 2) > -2(x + 4)$

$\Rightarrow\ 3x - 3 - x + 2 > -2x - 8$

$\Rightarrow\ 2x - 1 > -2x - 8$

$\Rightarrow\ 2x + 2x > -8 + 1$

$\Rightarrow\ 4x > -7$

$\Rightarrow x > -\dfrac{7}{4}$

Hence, $S.S = \{x | x > -\frac{7}{4}\}$

(viii). $\mathbf{2\frac{2}{3} + \frac{2}{3}(5x - 4) > -\frac{1}{3}(8x + 7)}$

Solution: As given $2\frac{2}{3} + \frac{2}{3}(5x - 4) > -\frac{1}{3}(8x + 7)$

$\Rightarrow\ \dfrac{8}{3} + \dfrac{2}{3}(5x - 4) > -\dfrac{1}{3}(8x + 7)$

Multiply by 3 (LCM)

$$\Rightarrow\ 3 \times \left(\frac{8}{3}\right) + 3 \times \left(\frac{2}{3}(5x - 4)\right) > 3 \times \left(-\frac{1}{3}(8x + 7)\right)$$

$\Rightarrow\ 8 + 2(5x - 4) > -(8x + 7)$

$\Rightarrow\ 8 + 10x - 8 > -8x - 7)$

$\Rightarrow\ 10x > -8x - 7$

$\Rightarrow\ 10x + 8x > -7$

$\Rightarrow\ 18x > -7$

$\Rightarrow\ x < -\dfrac{7}{18}$

Hence, $S.S = \{x | x < -\frac{7}{18}\}$

Q#2) Solve the following inequalities.

(i). $\mathbf{-4 < 3x + 5 < 8}$

Solution: As given $-4 < 3x + 5 < 8$

$\Rightarrow\ -4 - 5 < 3x < 8 - 5$

$\Rightarrow\ -9 < 3x < 3$

$\Rightarrow\ -\dfrac{9}{3} < \dfrac{3x}{3} < \dfrac{3}{3}$

$\Rightarrow\ -3 < x < 1$

Hence, $S.S = \{x | -3 < x < 1\}$

(ii). $-5 < \frac{4-3x}{2} < 1$

Solution: As given $-5 < \frac{4-3x}{2} < 1$

Multiply by 2

$\Rightarrow -10 < 4 - 3x < 2$

$\Rightarrow 10 - 4 < 4 - 3x - 4 < 2 - 4$

$\Rightarrow -14 < -3x < -2$

Multiply by -1 (inequality changes)

$\Rightarrow 14 > 3x > 2$

$\Rightarrow \frac{14}{3} > x > \frac{2}{3}$

Hence, $S.S = \{x | \frac{14}{3} > x > \frac{2}{3}$

(iii). $-6 < \frac{x-2}{4} < 6$

Solution: As given $-6 < \frac{x-2}{4} < 6$

$\Rightarrow -24 < x - 2 < 24$

$\Rightarrow -24 + 2 < x - 2 + 2 < 24 + 2$

$\Rightarrow -22 < x < 26$

Hence, $S.S = \{x | -22 < x < 26\}$

(iv). $3 \geq \frac{7-x}{2} \geq 1$

Solution: As given $3 \geq \frac{7-x}{2} \geq 1$

$\Rightarrow 6 \geq 7 - x \geq 2$

$\Rightarrow 6 - 7 \geq -x \geq 2 - 7$

$\Rightarrow -1 \geq -x \geq -5$

Multiply by -1

$\Rightarrow 1 \leq x \leq 5$

Hence, $S.S = \{x | 1 \leq x \leq 5\}$

(v). $3x - 10 \leq 5 < x + 3$

Sol: As given $3x - 10 \leq 5 < x + 3$

$3x - 10 \leq 5 \quad or \quad 5 < x + 3$

$\Rightarrow 3x \leq 5 + 10 \quad or \quad 5 - 3 < x$

$\Rightarrow 3x \leq 15 \quad or \quad 2 < x$

$\Rightarrow x \leq 5 \quad or \quad 2 < x$

$\Rightarrow 2 < x \quad or \quad x \leq 5$

$\Rightarrow 2 < x \leq 5$

Hence, $S.S = \{x | 2 < x \leq 5\}$

(vi). $-3 < \frac{x-4}{-5} < 4$

Solution: As given $-3 < \frac{x-4}{-5} < 4$

Multiply by -5

$\Rightarrow -5 \times (-3) > -5 \times \left(\frac{x-4}{-5}\right) < -5 \times (4)$

$\Rightarrow 15 < x - 4 < -20$

$\Rightarrow 15 + 4 < x < -20 + 4$

$\Rightarrow 19 > x > -16$

Hence, $S.S = \{x | -16 < x < 19\}$

(vii). $1 - 2x < 5 - x < 25 - 6x$

Solution : As given $1 - 2x < 5 - x < 25 - 6x$

$1 - 2x < 5 - x \quad or \quad 5 - x < 25 - 6x$

$\Rightarrow 1 - 5 < -x + 2x \quad or \quad -x + 6x < 25 - 5$

$\Rightarrow -4 < x \quad or \quad 5x < 20$

$\Rightarrow -4 < x \quad or \quad x < 4$

$\Rightarrow -4 < x < 4$

Hence, $S.S = \{x | -4 < x < 4\}$

(viii). $3x - 2 < 2x + 1 < 4x + 17$

Solution: As given $3x - 2 < 2x + 1 < 4x + 17$

$3x - 2 < 2x + 1 \quad or \quad 2x + 1 < 4x + 17$

$\Rightarrow 3x - 2x < 1 + 2 \quad or \quad 1 - 17 < 4x - 2x$

$\Rightarrow x < 3 \quad or \quad -16 < 2x$

$\Rightarrow x < 3 \quad or \quad -8 < x$

$\Rightarrow -8 < x \ or \quad x < 3$

$\Rightarrow -8 < x < 3$

Hence, $S.S = \{x | -8 < x < 3\}$

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MATHEMATICS 9th Science Group

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Chapter 9.

INTRODUCTION TO CO-ORDINATES GEOMETRY

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Contents

Coordinate Geometry

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane).

Distance Formula

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the coordinate plane where d is the length of the

line segment PQ. i.e. $|PQ| = d$ and given as

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXERCISE 9.1

Q#1) Find the distance between the following pairs of points.

(a) $A(9,2), B(7,2)$

Sol: As given $A(9,2), B(7,2)$

Using distance formula

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put $x_1 = 9, x_2 = 7, y_1 = 2$ and $y_2 = 2$

$|d| = \sqrt{(7-9)^2 + (2-2)^2}$

$|d| = \sqrt{(-2)^2 + (0)^2}$

$|d| = \sqrt{4}$

$|d| = 2$

(b) $A(2,-6), B(3,-6)$

Sol: As given $A(2,-6), B(3,-6)$

Using distance formula

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put $x_1 = 2, x_2 = 3, y_1 = -6$ and $y_2 = -6$

$|d| = \sqrt{(3-2)^2 + (2-2)^2}$

$|d| = \sqrt{(1)^2 + (0)^2}$

$|d| = \sqrt{1}$

$|d| = 1$

(c) $A(-8,1), B(6,1)$

Sol: As given $A(-8,1), B(6,1)$

Using distance formula

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put $x_1 = -8, x_2 = 6, y_1 = 1$ and $y_2 = 1$

$|d| = \sqrt{(6-(-8))^2 + (1-1)^2}$

$|d| = \sqrt{(6+8)^2 + (0)^2}$

$|d| = \sqrt{14^2}$

$|d| = 14$

(d) $A(-4, \sqrt{2}), B(-4,-3)$

Sol: As given $A(-4, \sqrt{2}), B(-4,-3)$

Using distance formula

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put $x_1 = -4, x_2 = -4, y_1 = \sqrt{2}$ and $y_2 = -3$

$|d| = \sqrt{(-4-(-4))^2 + \left(-3-\sqrt{2}\right)^2}$

$|d| = \sqrt{(-4+4)^2 + \left(3+\sqrt{2}\right)^2}$

$|d| = \sqrt{\left(3+\sqrt{2}\right)^2}$

$|d| = 3 + \sqrt{2}$

(e) $A(3,-11), B(3,-4)$

Sol: As given $A(3,-11), B(3,-4)$

Using distance formula

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put $x_1 = 3, x_2 = 3, y_1 = -11$ and $y_2 = -4$

$|d| = \sqrt{(3-3))^2 + (-4-(-11))^2}$

$|d| = \sqrt{(0)^2 + (-4+11)^2}$

$|d| = \sqrt{(7)^2}$

$|d| = 7$

(f) $A(0,0), B(0,-5)$

Sol: As given $A(0,0), B(0,-5)$

Using distance formula

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put $x_1 = 0, x_2 = 0, y_1 = 0$ and $y_2 = -5$

$|d| = \sqrt{(0-0)^2 + (-5-0)^2}$

$|d| = \sqrt{(0)^2 + (-5)^2}$

$|d| = \sqrt{5^2}$

$|d| = 5$

Q#2) Let P be the point on x-axis with x-coordinate a and Q be the point on y-axis with y-coordinate b as given below. Find the distance between P and Q.

(i) $a = 9, b = 7$

Sol: As Given $a = 9, b = 7$

$P(a, 0) = P(9, 0)$ and $Q(0, b) = Q(0, 7)$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0-9)^2 + (7-0)^2}$$

$$|PQ| = \sqrt{(-9)^2 + (7)^2}$$

$$|PQ| = \sqrt{81 + 49}$$

$$|PQ| = \sqrt{130}$$

(ii) $a = 2, b = 3$

Sol: As Given $a = 2, b = 3$

$P(a, 0) = P(2, 0)$ and $Q(0, b) = Q(0, 3)$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0-2)^2 + (3-0)^2}$$

$$|PQ| = \sqrt{(-2)^2 + (3)^2}$$

$$|PQ| = \sqrt{4 + 9}$$

$$|PQ| = \sqrt{13}$$

(iii) $a = -8, b = 6$

Sol: As Given $a = -8, b = 6$

$P(a, 9) = P(-8, 0)$ and $Q(0, b) = Q(0, 6)$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0-(-8))^2 + (6-0)^2}$$

$$|PQ| = \sqrt{(8)^2 + (6)^2}$$

$$|PQ| = \sqrt{64 + 36}$$

$$|PQ| = \sqrt{100} = 10$$

(iv) $a = -2, b = -3$

Sol: As Given $a = -2, b = -3$

$P(a, 9) = P(-2, 0)$ and $Q(0, b) = Q(0, -3)$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0-(-2))^2 + (-3-0)^2}$$

$$|PQ| = \sqrt{(2)^2 + (-3)^2}$$

$$|PQ| = \sqrt{4 + 9}$$

$$|PQ| = \sqrt{13}$$

(v) $a = \sqrt{2}, b = 1$

Sol: As Given $a = \sqrt{2}, b = 1$

$P(a, 9) = P(\sqrt{2}, 0)$ and $Q(0, b) = Q(0, 1)$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{\left(0-\sqrt{2}\right)^2 + (1-0)^2}$$

$$|PQ| = \sqrt{\left(\sqrt{2}\right)^2 + (1)^2}$$

$$|PQ| = \sqrt{2 + 1}$$

$$|PQ| = \sqrt{3}$$

(vi) $a = -9, b = -4$

Sol: As Given $a = -9, b = -4$

$P(a, 9) = P(-9, 0)$ and $Q(0, b) = Q(0, -4)$

Using distance formula

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|PQ| = \sqrt{(0-(-9))^2 + (-4-0)^2}$$

$$|PQ| = \sqrt{(9)^2 + (-4)^2}$$

$$|PQ| = \sqrt{81 + 16}$$

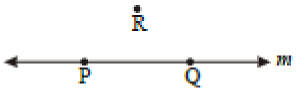
$$|PQ| = \sqrt{97}$$

Collinear or Non-collinear Points in the Plane

Two or more than two points which lie on the same straight line are called collinear points with respect to that line; otherwise they are called non-collinear.

Let m be a line, then all the points on line m are collinear.

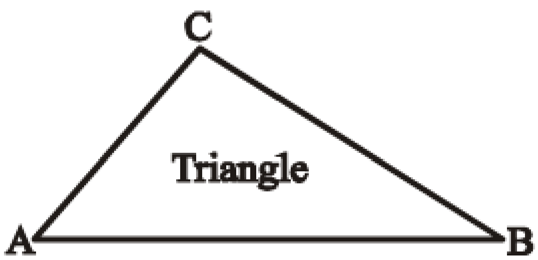
In the given figure, the points P and Q are collinear with respect to the line m and the points P and R are not collinear with respect to it.



Triangle

A closed figure in a plane obtained by joining three non-collinear points is called a triangle.

In the triangle ABC the non-collinear points $A, B\ and\ C$ are the three vertices of the triangle ABC. The line segments $AB, BC\ and\ CA$ are called sides of the triangle.



(i) Equilateral Triangle

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle
.

(ii) An Isosceles Triangle

An isosceles triangle PQR is a triangle which has two of its sides with equal length while the third side has a different length.

(iii) Right Angle Triangle

A triangle in which one of the angles has measure equal to 90^0 is called a right angle triangle.

(iv) Scalene Triangle

A triangle is called a scalene triangle if measures of all the three sides are different.

Square

A square is a closed figure in the plane formed by four non-collinear points such that lengths of all sides are equal and measure of each angle is 90^o.

Rectangle

A figure formed in the plane by four non-collinear points is called a rectangle if,

(i) Its opposite sides are equal in length;

(ii) The angle at each vertex is of measure 90^o.

Parallelogram

A figure formed by four non-collinear points in the plane is called a **parallelogram** if

(i) its opposite sides are of equal length

(ii) its opposite sides are parallel

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points in the plane and $M(x, y)$ be a mid-point of points P and Q on the line-segment PQ is given as

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

EXERCISE 9.3

Q#1) Find the mid-point of the line segment joining each of the following pairs of points

(a) $A(9,2), B(7,2)$

Sol: As given $A(9,2), B(7,2)$

Using Mid-point formula

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put $x_1 = 9, x_2 = 7, y_1 = 2\ and\ y_2 = 2$

$$Mid-point\ of\ PQ = M\left(\frac{9+7}{2}, \frac{2+2}{2}\right)$$

$$Mid-point\ of\ PQ = M\left(\frac{16}{2}, \frac{4}{2}\right)$$

$$Mid-point\ of\ PQ = M(8,2)$$

(b) $A(2,-6), B(3,-6)$

Sol: As given $A(2,-6), B(3,-6)$

Using Mid-point formula

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put $x_1 = 2, x_2 = 3, y_1 = -6\ and\ y_2 = -6$

$$Mid-point\ of\ PQ = M\left(\frac{2+3}{2}, \frac{-6-6}{2}\right)$$

$$Mid-point\ of\ PQ = M\left(\frac{5}{2}, \frac{-12}{2}\right)$$

$$Mid-point\ of\ PQ = M(2.5,-6)$$

(c) $A(-8,1), B(6,1)$

Sol: As given $A(-8,1), B(6,1)$

Using Mid-point formula

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

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Put $x_1 = -8, x_2 = 6, y_1 = 1$ *and* $y_2 = 1$

$$Mid - point\ of\ PQ = M\left(\frac{-8+6}{2}, \frac{1+1}{2}\right)$$

$$Mid - point\ of\ PQ = M\left(\frac{-2}{2}, \frac{2}{2}\right)$$

$$Mid - point\ of\ PQ = M(-1, 1)$$

(d) $A(-4, 9), B(-4, -3)$

Sol: As given $A(-4, \sqrt{2}), B(-4, -3)$

Using Mid-point formula

$$Mid - point\ of\ PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put $x_1 = -4, x_2 = -4, y_1 = 9$ *and* $y_2 = -3$

$$Mid - point\ of\ PQ = M\left(\frac{-4-4}{2}, \frac{9-3}{2}\right)$$

$$Mid - point\ of\ PQ = M\left(\frac{-8}{2}, \frac{6}{2}\right)$$

$$Mid - point\ of\ PQ = M(-4, 3)$$

(e) $A(3, -11), B(3, -4)$

Sol: As given $A(3, -11), B(3, -4)$

Using Mid-point formula

$$Mid - point\ of\ PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put $x_1 = 3, x_2 = 3, y_1 = -11$ *and* $y_2 = -4$

$$Mid - point\ of\ PQ = M\left(\frac{3+3}{2}, \frac{-11-4}{2}\right)$$

$$Mid - point\ of\ PQ = M\left(\frac{6}{2}, \frac{-15}{2}\right)$$

$$Mid - point\ of\ PQ = M(3, -7.5)$$

(f) $A(0, 0), B(0, -5)$

Sol: As given $A(0, 0), B(0, -5)$

Using Mid-point formula

$$Mid - point\ of\ PQ = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put $x_1 = 0, x_2 = 0, y_1 = 0$ *and* $y_2 = -5$

$$Mid - point\ of\ PQ = M\left(\frac{0+0}{2}, \frac{0-5}{2}\right)$$

$$Mid - point\ of\ PQ = M\left(\frac{0}{2}, \frac{-5}{2}\right)$$

$$Mid - point\ of\ PQ = M(0, 2.5)$$

REVIEW EXERCISE

Q#2) Answer the following, which is true and which is false.

(i) A line has two end points…F …..

(ii) A line segment has one end point…F …..

(iii) A triangle is formed by three collinear points. …F

(iv) Each side of a triangle has two collinear vertices…T…

(v) The end points of each side of a rectangle are collinear…T ….

(vi) All the points that lie on the x-axis are collinear…T …

(vii) Origin is the only point collinear with the points of both the axes separately. …T..